19. NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS

Problem statement: Solution of equation: $y' = f(x, y),\ y(x_1) = y_1$

Solution: (i) analytical: function $y(x)$

(ii) numerical: sequence $\{x_i, y_i\}$, $i = 2, 3, \cdots$

COMMANDS

SYMS
DSOLVE
PRETTY
EZPLOT

ODE23
HOLD ON
HOLD OFF
PLOT

19.1 Symbolic Solution

Characteristics:

1. Symbolic solution is not always possible
2. Substitution allows conversion to numerical solution

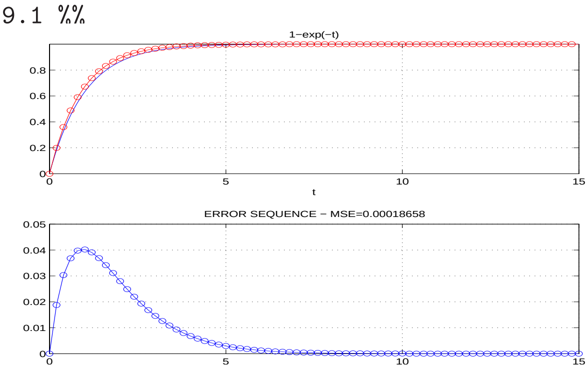
```
%%% Example 19.1: Ordinary Differential Equations - Initial-Value Problem
%%% Equation: y'+y=1, y(x1)=0; for x1=0 in range <x1, xk>
  clear; delete(get(0,'children')); syms t
  x1=0; xk=15;
  ys=dsolve('Dy+y=1','y(0)=0','t');
  subplot(2,1,1); ezplot(ys,[x1 xk]); grid on; axis tight
```

19.2 Numeric Solution

Principle of Euler method for the Initial Value Problem:

1. Approximate solution: $y_i = y_{i-1} + h\, f(x_{i-1}, y_{i-1})$, where $x_i = x_{i-1} + h$, $i = 2, 3, \cdots$ and h is a chosen step
2. Step value can affect accuracy and stability

```
%%% Example 19.2: Ordinary Differential Equations - Initial-Value Problem
%%% Equation: y'=1-y, y(x1)=0; %%%%%%%%%%%%%% ... Cont of Ex.19.1 %%
  h=input('Step (=0.2): ');
  x(1)=x1; y(1)=0; N=(xk-x1)/h+1;
  for i=2:N
      x(i)=x(i-1)+h;
      y(i)=y(i-1)+h*(1-y(i-1)); end
  hold on; plot(x,y,'-or'); axis tight; hold off
  subplot(2,1,2); e=y-subs(ys,t,x); S=sumsqr(e)/length(e);
    plot(x,e,'-o');  grid on
    title(['ERROR SEQUENCE - MSE=',num2str(S)]);
```

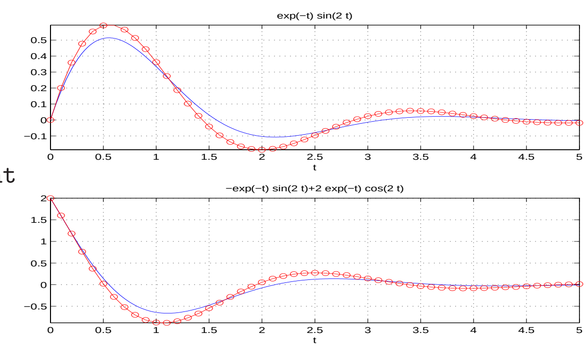


19.3 Solution of System of Equations

Euler method application for solution of a system: $\mathbf{y}' = \mathbf{f}(x, \mathbf{y}),\ \mathbf{y}(x_1) = \mathbf{y}^{(1)}$

1. Vector solution: $\mathbf{y}^{(i)} = \mathbf{y}^{(i-1)} + h\, \mathbf{f}(x_{i-1}, \mathbf{y}^{(i-1)})$, where $x_i = x_{i-1} + h$, $i = 2, 3, \cdots$ and h is a chosen step
2. Step value can affect accuracy and stability

```
%%% Example 19.3: System of Ordinary Differential Equations - Initial-Value Problem
% y''+2y'+5y=0, y(x1)=0; y'(x1)=2 for x1=0 in range <x1, xk>
%%%%%% A. Symbolic Solution %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
  clear; delete(get(0,'children')); syms t; x1=0; xk=5;
  ys=dsolve('D2y+2*Dy+5*y=0','y(0)=0','Dy(0)=2');
    pretty(simplify(ys)); ezplot(ys,[x1 xk]); axis tight
  subplot(2,1,1); ezplot(ys,[x1 xk]); grid on; axis tight
  subplot(2,1,2); ezplot(diff(ys),[x1 xk]); grid on; axis tight
%%%%%% B. Numeric Solution: Euler Method %%%%%%%%%%%%%%%%
% Equation: y1'=-2*y1-5*y2; y1(x1)=2
%           y2'=y1;         y2(x1)=0;
  h=input('Step (=0.1): ');
  x(1)=x1; y1(1)=2; y2(1)=0; N=(xk-x1)/h+1;
  for i=2:N
      x(i)=x(i-1)+h; y1(i)=y1(i-1)+h*(-2*y1(i-1)-5*y2(i-1));
                     y2(i)=y2(i-1)+h*y1(i-1); end
  subplot(2,1,1); hold on; plot(x,y2,'-or'); axis tight; hold off
  subplot(2,1,2); hold on; plot(x,y1,'-or'); axis tight; hold off
```



EXAMPLES 19

19.1 Evaluate symbolic solution of a selected system of differential equations with given initial conditions

19.2 Evaluate numeric solution of a selected system of differential equations with given initial conditions