12. SYSTEMS OF LINEAR EQUATIONS

Problem statement: Solution of system of equations

$$\begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,N} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,N} \\ & & \cdots & \\ a_{N,1} & a_{N,2} & \cdots & a_{N,N} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_N \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \cdots \\ b_N \end{pmatrix} \quad \Leftrightarrow \quad \mathbf{A}_{N,N}\, \mathbf{x}_{N,1} = \mathbf{b}_{N,1}$$

COMMANDS

SYMS
SOLVE
SUBS

SIMPLE
PRETTY

INV
DIAG

SPARSE
FULL
SPY
GPLOT

12.1 Symbolic Solution

Characteristics:

1. Possibilities of simplification of symbolic solution
2. Substitution allows conversion to numerical solution

```
%%% Example 12.1: Symbolic solution of linear equations
>>    syms a11 a12 a21 a22 b1 b2 x1 x2    % Definition of symbolic variables
>>    A=[a11 a12; a21 a22]; b=[b1; b2]; x=A\b; pretty(simple(x))
%%% Alternative
>>    EQ=A*[x1; x2]-b; SOLUTION=solve(EQ(1),EQ(2));
>>      s1=SOLUTION.x1; pretty(simple(s1))
>>      s2=SOLUTION.x2; pretty(simple(s2))
>>    s=subs(x,{a11 a12 a21 a22 b1 b2},{1 1 2 1 3 4}) % Substitution
```

12.2 Numerical Solution

12.2.1 Finite Methods

Mathematical background of Gauss-Jordan method:

$$\begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,N} & b_1 \\ a_{2,1} & a_{2,2} & \cdots & a_{2,N} & b_2 \\ & & \cdots & & \\ a_{N,1} & a_{N,2} & \cdots & a_{N,N} & b_N \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & \cdots & 0 & x_1 \\ 0 & 1 & \cdots & 0 & x_2 \\ & & \cdots & & \\ 0 & 0 & \cdots & 1 & x_N \end{pmatrix}$$

System definition

$$\mathbf{A} = \begin{pmatrix} 4 & -1 & 1 \\ 1 & 6 & 2 \\ -1 & -2 & 5 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 4 \\ 9 \\ 2 \end{pmatrix} \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

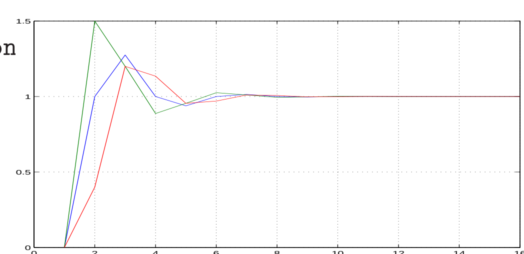
```
%%% Example 12.2: Gauss-Jordan method
>>  A=[4 -1 1;1 6 2;-1 -2 5]; b=[4 9 2]'; S=[A,b]; [m,n]=size(S);
>>  for i=1:m
>>    [Max,j]=max(abs(S(:,i)));  % Localization of the maximum element of i-th column
>>    S([j i],:)=S([i j],:);     % Row exchange
>>    S(i,:)=S(i,:)/S(i,i);      % Division of the i-th row by the diagonal element
>>    for ii=[1:i-1,i+1:m]       % Elimination
>>      S(ii,:)=S(ii,:)-S(ii,i)*S(i,:);
>>    end
>>  end; x=S(:,4)
```

12.2.2 Iterative Methods

Mathematical background:

$$\begin{pmatrix} x_1^{(k)} \\ x_2^{(k)} \\ \cdots \\ x_N^{(k)} \end{pmatrix} = \left(\begin{pmatrix} b_1 \\ b_2 \\ \cdots \\ b_N \end{pmatrix} - \begin{pmatrix} 0 & a_{1,2} & \cdots & a_{1,N} \\ a_{2,1} & 0 & \cdots & a_{2,N} \\ & & \cdots & \\ a_{N,1} & a_{N,2} & \cdots & 0 \end{pmatrix} \begin{pmatrix} x_1^{(k-1)} \\ x_2^{(k-1)} \\ \cdots \\ x_N^{(k-1)} \end{pmatrix} \right) ./ \begin{pmatrix} a_{1,1} \\ a_{2,2} \\ \cdots \\ a_{N,N} \end{pmatrix} \quad \Rightarrow \quad \mathbf{x}^{(k)} = (\mathbf{b} - (\mathbf{A} - diag(diag(\mathbf{A}))\, \mathbf{x}^{(k-1)})./diag(\mathbf{A})$$

```
%%% Example 12.3: Iterative solution
     x=[0 0 0]';                       % Initial estimate of the solution
     delta=0.0001; MAX=50; Z=x'; % Precission, iterations
     for i=1:MAX
       x=(b-(A-diag(diag(A)))*x)./diag(A);
       Z=[Z;x']; [m,n]=size(Z);
       if max(abs(Z(m,:)-Z(m-1,:)))<delta, break, end
     end; x, plot(Z)
```



EXAMPLES 12

12.1 Evaluate symbolic and numeric solution of a selected system of linear equations

12.2 Study methods of solution of sparse systems of linear equations