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Introduction

Digital signal processing (DSP) represents a general interdisciplinary area [36, 30, 3, 41, 35] based upon numerical or symbolical analysis of one-dimensional or multi-dimensional data sets that may stand for any physical, engineering, biomedical, technological, biological, acoustic, seismic or economical variable measured or observed with a given sampling period. Selected applications and goals of their processing are presented in Fig. 1.1. Even though applications cover completely different areas the mathematical background of their analysis is very close allowing processing of vectors, matrices or multi-dimensional arrays of observed data in a general way. Digital signal processing methods thus form an integrating platform for many diverse research branches.

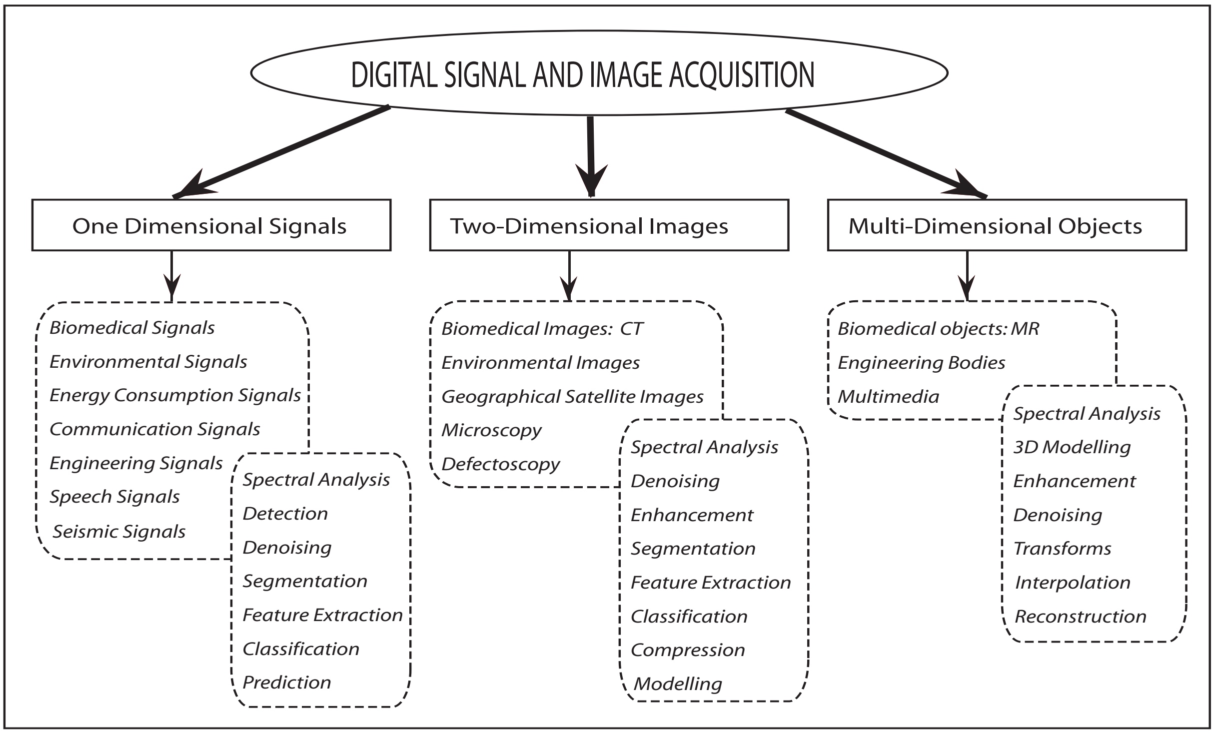

FIGURE 1.1. Fundamental applications of one-dimensional and multi-dimensional signal processing and selected goals of their analysis

Fundamental mathematical methods of signal, image and multi-dimensional objects processing in the space and frequency domains are summarized in Fig. 1.2 and they include the following main topics

- Space domain deterministic processing
- Probabilistic signal processing
- Space domain adaptive processing
- Space-Frequency analysis
- Space-Scale analysis

Selected mathematical methods presented in this figure cover basic numerical methods [4], statistical methods, adaptive methods including neural networks, discrete Fourier transform and discrete wavelet transform. Goals of signal analysis cover the estimation of its characteristic parameters either in the *original* or in the *transform* domain. In some cases of signal processing *deterministic methods* may be applied but in many applications *statistical* and *adaptive methods* [12, 33] must be used to compensate for the incomplete knowledge of the real system time variations.

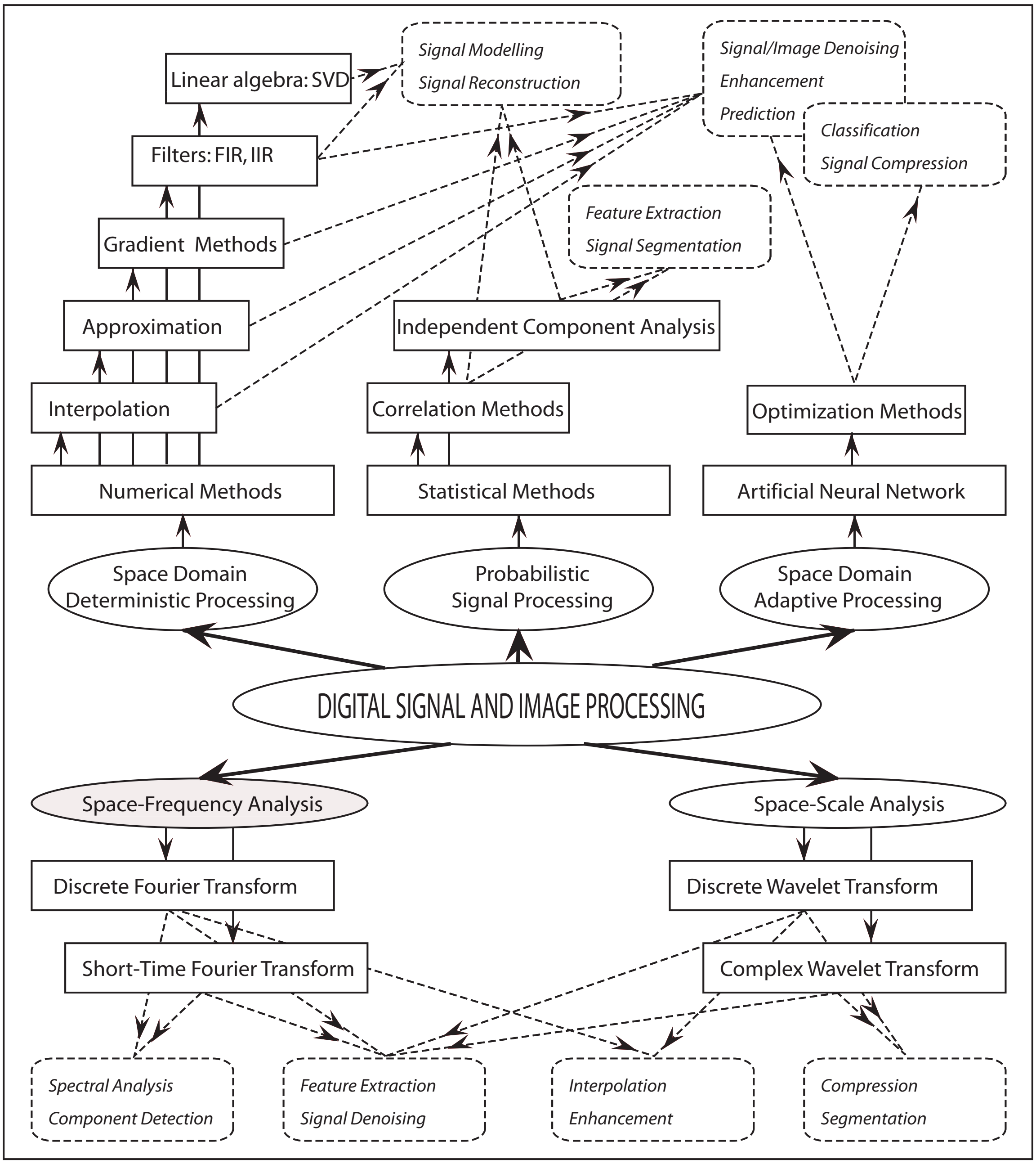

FIGURE 1.2. Fundamental mathematical methods of digital signal and image processing both in the space and frequency domains and associated goals of their applications

1.1 Historical Notes

The mathematical fundamentals of digital signal and image processing methods are based upon *numerical analysis* [1] that predates the invention of modern computers by many centuries using works of famous mathematicians including that of Isaac Newton (1643-1727), Joseph Louis Lagrange (1736-1813) and Leonhard Euler (1707-1783). The *matrix theory* introduced in the middle of the 19th century incorporating ideas of Gottfried Wilhelm Leibnitz (1646-1716) and Carl Friedrich Gauss (1777-1855) forms now one of its basic mathematical tools as well.

Isaac Newton
(January 1,1643-March 31,1727)

Jean Baptiste Joseph Fourier
(March 21,1768-May 16,1830)

Carl Friedrich Gauss
(April 30,1777-February 23,1855)

The theory of digital signal processing is in many cases closely connected with the *Fourier representation* of functions suggested in 1822 [9] by Jean Baptiste Joseph Fourier (1786-1830) and on the *method of the least squares* presented independently by Carl Friedrich Gauss [10] and Adrien-Marie Legendre (1752-1833) [21] in the beginning of the 19th century.

Basic mathematical methods were later extended to many fields including the estimation theory and stochastic processes introduced by Norbert Wiener (1894-1964) in 1949 [44] and Rudolf E. Kalman (1930-) [19] with applications in various areas covering adaptive filtering problems and spectrum analysis. Many algorithms use properties of the discrete Fourier transform and their implementation is enabled by its fast version published by James Cooley (1926-) and John Tukey (1915-2000) [7] in 1965. The latest research of wavelet transform using the first known wavelet proposed by Alfred Haar (1885-1933) in 1909 is based upon research of Ingrid Daubechies (1954-) published in 1992 [8].

Adrien-Marie Legendre
(September 18,1752-January 10,1833)

1.2 Applications

Basic methods and applications of digital signal and image processing methods summarized in Fig. 1.1 cover [40, 12, 3, 41, 14]

- *Spectrum estimation* giving the distribution of power over frequency and enabling in this way to distinguish characteristic signal components important for analysis and further processing [13] in biomedicine, chemistry, seismology, communications and control systems,
- *Digital filtering* originally used to eliminate undesirable frequency components or to reduce noise in communications, multimedia systems, control structures or biomedical data [29, 16, 17] and to improve possibilities of acoustic and image processing,
- *Correlation techniques* enabling comparison of signals representing in many cases sampled physical quantity.

Classical applications mentioned above has been substantially enlarged by the use of adaptive filters giving ability to operate in an unknown environment on discrete signals representing any physical or technological variable. General objectives of such a processing may result in

- *System identification* substantial for mathematical modelling in science or engineering and including *parameter estimation* and *inverse modelling* as well,
- *Signal detection* and *prediction* enabling to find out the useful information in a given sequence and to forecast its behaviour,
- *Interference cancelling* used to reject undesirable signal components for further signal analysis and processing.

In case of general systems time-varying models are obtained and methods used for adaptive processing are closely related to that of computational intelligence and neural networks.

1.3 Algorithmic Tools

Computer experiments suggested in this text may be transformed into any computer language but all presentations described bellow were realized in the MATLAB software package [11, 26, 27, 6] allowing a very simple realization of all methods in form very close to the original matrix notation. There are many books devoted to signal and image processing using this computational environment [38, 14, 37, 28].