WAVELET USE FOR IMAGE RESTORATION

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Abstract: The interdisciplinary area of digital signal and image processing forms a basis for de-noising, enhancement, recovery and classification of biomedical images. The paper is devoted to the wavelet transform as a general mathematical tool and its use for image decomposition and reconstruction to recover its corrupted regions. The main part of the paper is devoted to image decomposition, thresholding and reconstruction to reject specific image components. A special attention is paid to recovery of image regions using iterated wavelet decomposition and reconstruction. All algorithms are verified for simulated images and then applied to biomedical images obtained by magnetic resonance. The Matlab programming environment has been used to realize the algorithms and to obtain the presented results.

Keywords: Wavelet Transform, Signal Processing, Image Analysis, Signal Decomposition and Reconstruction, De-noising

1. Introduction

Signal and image processing became an integral part of many engineering disciplines in the last century allowing to find similar mathematical description of diverse applications including biomedical image analysis, environmental signal processing, control system modelling, speech analysis and data forecasting. In this way it forms an interdisciplinary basis for physics and mathematics using information engineering and modern information technologies [2,4,6,7,8].

Mathematical methods of signal and image analysis are based in many cases on the one-dimensional or two-dimensional discrete Fourier transform or on the wavelet transform [1,3] allowing either time-frequency or time-scale signal analysis. The following signal and image processing use both linear methods including FIR filters and non-linear methods based upon artificial neural networks [1,9,10] using various optimization methods.

2. Fundamentals of Wavelet Transform

Wavelet transforms (WT) provide the alternative to the short-time Fourier transform (STFT) for non-stationary signal analysis [4,9]. Both STFT and WT result in signal decomposition into two-dimensional function of time and frequency respectively scale. The basic difference between these two transforms is in the construction of the window function which has a constant length in the case of the STFT (including rectangular, Blackman and other window functions) while in the case of the WT wide windows are applied for low frequencies and short windows for high frequencies to ensure constant time-frequency resolution. Local and global signal analysis can be combined in this way.

Wavelet functions used for signal analysis are derived from the initial basic (mother) function $h(t)$ forming the set of functions

$$h_{m,k}(t) = \frac{1}{\sqrt{a}} h\left(\frac{1}{a}(t-b)\right) = \frac{1}{\sqrt{2^m}} h\left(2^{-m} t - k\right) \qquad (1)$$

for discrete parameters of dilation $a = 2^m$ and translation $b = k2^m$. Wavelet dilation corresponds to spectrum compression according to Fig. 1. The most common choice includes Daubechies wavelets even though their frequency characteristics stand for approximation of band-pass filters only. On the other hand harmonic wavelets introduced in [5] can have broader application in many engineering problems owing to their very attractive spectral properties.

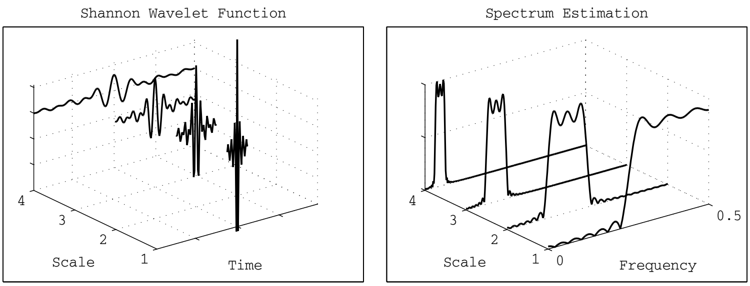

Fig. 1. Spectral analysis of selected wavelet function presenting relation between time dilation and the corresponding spectrum compression

3. Signal and Image De-Noising

Information about signals resulting from a selected process can be based upon signal decomposition by a given set of wavelet functions into separate levels or scales resulting in the set of wavelet transform coefficients. These values can be used for signal compression, signal analysis, segmentation and in the case that these coefficients are not modified they allow the following perfect signal reconstruction. In the case that only selected levels of signal decomposition are used or wavelet transform coefficients are processed it is possible to extract signal components or to reject its undesirable parts.

Using the threshold method introduced by [1,9] it is further possible to reject noise and to enlarge signal to noise ratio. The de-noising algorithm assumes that the signal contains low frequency components and it is corrupted by the additive Gaussian white noise with its power much lower than power of the analyzed signal. The whole method consists of the following steps:

- Signal decomposition using a chosen wavelet function up to the selected level and evaluation of wavelet transform coefficients
- The choice of threshold limits for each decomposition level and modification of its coefficients
- Signal reconstruction from modified wavelet transform coefficients

Results of this process depend upon the proper choice of wavelet functions, selection of threshold limits and their use.

The application of threshold limits for a modification of the wavelet coefficients $\{c(k)\}_{k=0}^{N-1}$ includes two basic approaches. The use of the soft thresholding formula for a chosen thresholding value δ results in the evaluation of new coefficients by the following relation

$$\overline{c_s}(k) = \begin{cases} \text{sign } c(k)\left(|c(k)| - \delta\right) & \text{if } |c(k)| > \delta \\ 0 & \text{if } |c(k)| \leq \delta \end{cases} \tag{2}$$

The hard thresholding method results in the following values of coefficients

$$\overline{c_h}(k) = \begin{cases} \text{sign } c(k) & \text{if } |c(k)| > \delta \\ 0 & \text{if } |c(k)| \leq \delta \end{cases} \tag{3}$$

These two methods can be applied both for one-dimensional and two-dimensional signals. An example of a simulated image decomposition and de-noising is presented in Fig. 2.

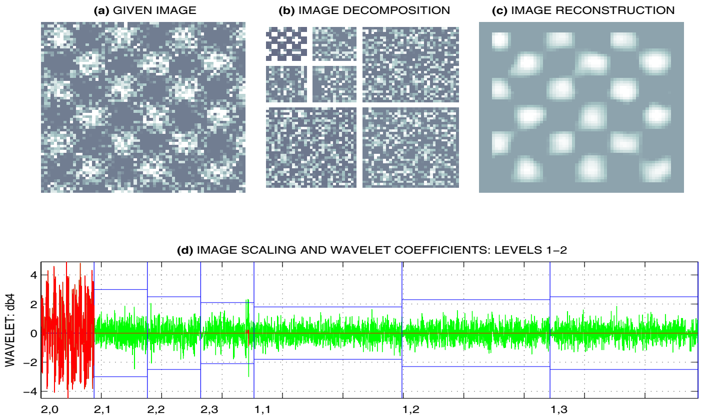

Fig. 2. Simulated image denoising presenting **(a)** given image, **(b)** image decomposition into two levels, **(c)** image reconstruction and **(d)** wavelet coefficients soft-thresholding

4. Wavelet Image Restoration

There are many possibilities of filling-in missing or corrupted image blocks (regions). The goal of this task sufficiently solves the iterative wavelet interpolation method designed for restoration of corrupted or missing image regions.

We can view a sequence of lost samples as the result of a particular noise process acting on the original signal. However, unlike the traditional case, this noise process is not uncorrelated with the original signal. The basic intuition behind denoising tries to keep transform coefficients of high PSNR (Peak Signal-to-Noise Ratio) while zeroing out coefficients having lower PSNR. Our primary assumption in this algorithm is that the transformation used to generate the wavelet transform coefficients mostly ensures that if vector **c** is hard-thresholded by Eq. (3) to zero with the threshold limit δ equal to the variance of signal noise σ_e, then this procedure removes more noise than signal.

This algorithm makes changes just to the lost sequence of samples by the transform coefficients hard-thresholded to zero. When the value of the lost samples is changed, we can continue to evaluate these samples again. Input signal for the wavelet decomposition, hard-thresholding, and backward wavelet reconstruction, is a result of the previous iteration. The algorithm is repeated until the SSE (Sum of Squared Errors) value between the recovered and the original signal is acceptably low or required PSNR value is achieved.

This technique has been applied to 2-D signals represented by real magnetic resonance images of the human brain. The only difference is that the wavelet decomposition is implemented to rows at first followed by the same algorithm applied to columns.

The proposed algorithm has been applied to the MR image of the human brain. Fig. 3 presents the wavelet decomposition of the original corrupted MR image (see Fig. 3(a)) into one decomposition level Fig. 3(b) using the Daubechies wavelet function of the 8th order. Fig. 3(d) shows the wavelet coefficients and modified, i.e. hard-thresholded wavelet coefficients. Recovered MR image (after the first iteration) can be seen in Fig. 3(c).

The final recovered MR image with low acceptable SSE and high PSNR has been obtained after 350 iterations of the iterative wavelet interpolation algorithm. Fig. 4(a) presents the original corrupted image and Fig. 4(b) recovered image. Evolution of the PSNR and SSE values during the whole recovery process is shown in Fig. 4(c),(d).

The proposed method based upon the fundamental principle introduced in [2] consists of the following iteration steps:

- Image wavelet decomposition into selected level and hard-thresholding of resulting coefficients
- Image reconstruction and transformation of the resulting image to preserve image values outside its corrupted regions creating the new image for the further iteration step

5. Results

As the most efficient method of the MR image denoising the wavelet Symmlet of the 4th order has been used here for the decomposition into two levels. The best result of the MR image recovery has been obtained using the Daubechies wavelet function of the 8th order (see Table 1) for the wavelet decomposition into one level and 350 steps. Coefficients in the wavelet domain have been modified by hard-thresholding before image reconstruction.

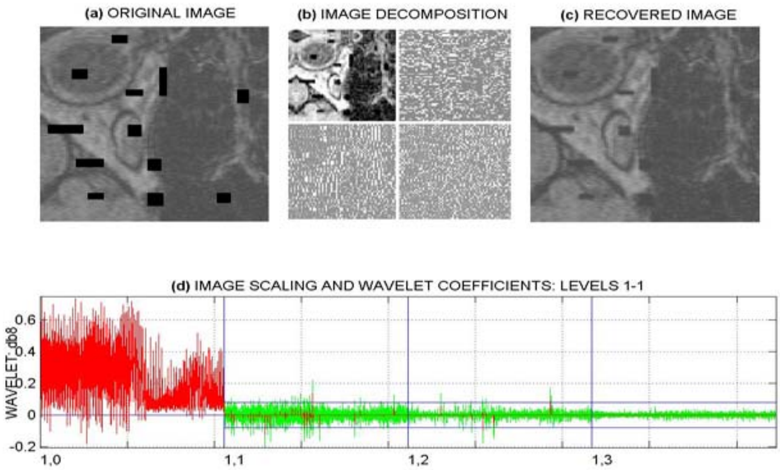

Fig. 3. Recovery of the real MR image presenting **(a)** given image, **(b)** image decomposition into the first levels, **(c)** image reconstruction and **(d)** wavelet coefficients hard-thresholding

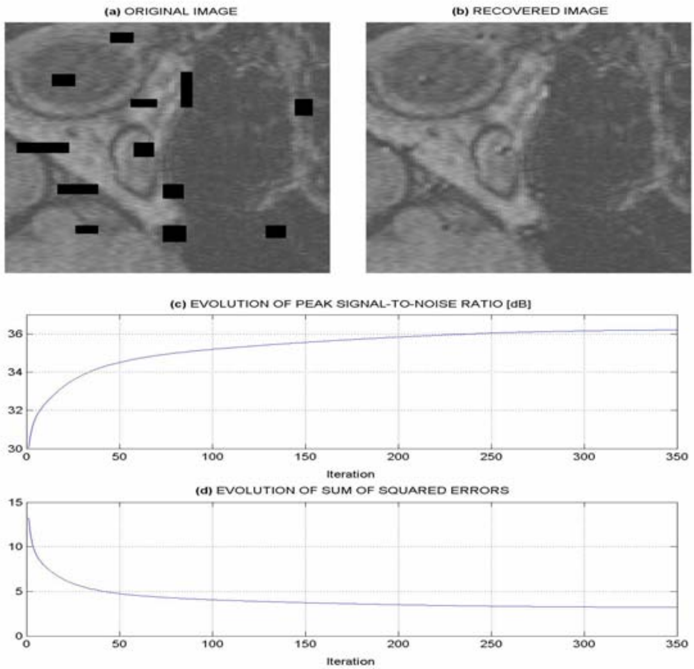

Fig. 4. Recovery of the real MR image using the iterated wavelet interpolation method presenting **(a)** given corrupted image, **(b)** recovered image (after 350 iterations), **(c)** evolution of the Peak Signal-to-Noise Ratio (PSNR) value, and **(d)** evolution of the Sum of Squared Errors (SSE) during the iteration process

Table 1. Peak Signal-to-Noise Ratio (PSNR) and Sum of Squared Errors (SSE) of the real MR image with corrupted regions (PSNR1, SSE1) and the same image after the recovery process (PSNR2, SSE2) reconstructed by the selected wavelet functions

Wavelet function	PSNR1 [dB]	PSNR2 [dB]	SSE1	SSE2
Haar Wavelet	29.078	30.143	16.6075	12.9957
Daubechies of the 2nd order		33.118		6.5512
Daubechies of the 4th order		34.715		4.5353
Daubechies of the 8th order		36.200		3.2219
Symmlet of the 2nd order		33.118		6.5512
Symmlet of the 4th order		35.657		3.6511
Symmlet of the 8th order		35.978		3.3912

6. Conclusion

There are many possibilities of restoration of corrupted image regions and image denoising. The paper presents selected algorithms using wavelet transform to achieve this goal. The advantage of the wavelet transform is in its ability to distinguish more frequency bands (number of decomposition levels) and setting of different threshold limits for each level. Therefore WT provides a very efficient tool for an image components reconstruction. The next improvement of the method can be in distribution of the lost sequence of samples into a few layers and hard-thresholding applied layer by layer. This should increase the edge sensitivity of the reconstruction.

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