Measurement Uncertainty Evaluation Using Monte Carlo Method

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Abstract: The Guide to the Expression of Uncertainty in Measurement (GUM) provides internationally agreed approaches and recommendations for the evaluation of uncertainty in metrology (1). One of these approaches is the one based on the principle of propagation of distributions. This principle determines the probability distribution for an output quantity from the probability distributions assigned to the input quantities on which the output quantity depends, and it can be implemented numerically using Monte Carlo method (2). So evaluation procedure of this method was realized using popularly held standard spreadsheet software Microsoft Excel.

Keywords: measurement uncertainty, propagation of distributions, Monte Carlo, spreadsheet

1 SOFTWARE DESCRIPTION

The probability density function (PDF) of a quantity determines the probability by which a certain value of this quantity is observed in a random measurement. Monte Carlo method is a practical numerical tool which simulates generally an unlimited number of unique measurements by random sampling from the known PDFs of all input quantities and propagates their distributions "through" the measurement model (i.e. mathematical relation among the input quantities and the output quantity). In accordance with the law of propagation of distributions, the distribution of simulated measurements results reflects PDF of the output quantity and also the measurement uncertainty.

Realized software Measurement Uncertainty is a spreadsheet application which implements Monte Carlo measurement uncertainty evaluation procedure for univariate, explicit, real measurement model $Y = f(\boldsymbol{X})$, in which Y, the output quantity, is the measurand, $\boldsymbol{X}$ is set of generally N number of input quantities and f is formula of measurement function. The procedure is as follows (3, 4):

1. Select the number M of Monte Carlo trials to be made.
2. Generate M vectors by random sampling from the PDFs for the (set of N) input quantities.
3. For each vector, evaluate the model to give the corresponding value of the output quantity.
4. From model values, calculate the estimate of the output quantity and the associated standard uncertainty as the (arithmetic) mean and standard deviation of the model values.
5. From model values, form the approximating PDF for the output quantity.
6. Sort the model values into non-decreasing order and use the sorted values to provide a discrete representation of the distribution function for the output quantity.

Note: Measurement model of realized spreadsheet application supposes up ten input quantities (i.e. $N <= 10$) with normal distribution and number of Monte Carlo trials greater than 1 000 and less than 100 000 (i.e. $M \in \langle 1\,000, 1\,000\,000 \rangle$).

Note: Evaluation procedure of realized spreadsheet application do not calculate coverage interval for the output quantity (which is generally possible from required coverage probability in accordance with GUM documents (1, 2)).

Figure 1 shows implemented procedure in the diagram.

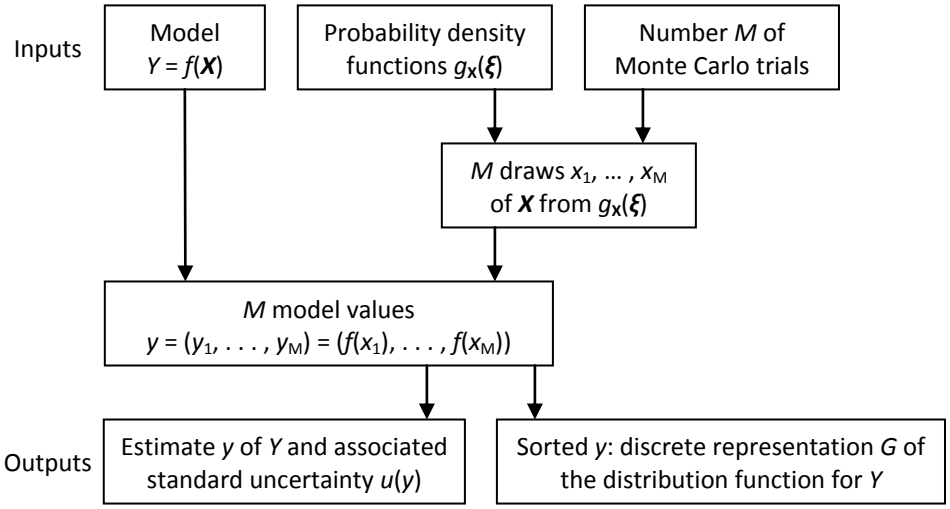


Fig. 1 Flow chart of Monte Carlo procedure implemented in the spreadsheet application

2 SOFTWARE REQUIREMENTS

Spreadsheet application is the file *MeasurementUncertainty.xltm* as MS Excel 2007 and MS Excel 2010 workbook that contains macros (series of VBA commands). Therefore, the spreadsheet application requires installation of MS Excel of some of mentioned versions and enabled macros (permanently in Trust Center or temporarily in Message Bar when you create new workbook – see on-line help for Office 2007 and Office 2010).

3 WORKING WITH SOFTWARE

Essentially, the file *MeasurementUncertaintyMCS.xltm* is the workbook template (the file with XLTM extension), i.e. the model that serves as the basis of other workbooks. If user wants to plug input data and to get results of uncertainty analysis he must create a new workbook based on the template and then manipulates only with new workbook.

Note: The new workbook copies content, formatting, VBA codes and customized user interface from the template, so the template remains unchanged. However, if user accidentally changes the template the full application functionality cannot be provided and the user must download the template again.

Way to work with the new workbook is a simple: firstly, it is possible to plug in data, and secondly, it is needful to enter the formula of the measurement model to obtain measurement evaluation. Each new workbook generates worksheets that can be handled in a standard way (Chap. 4.2 and 4.3).

3.1 USER INTERFACE

The new workbook based on the template has customized Ribbon interface with the new tab *Measurement Uncertainty* that contains these controls (Fig. 2):

- *New Input Data*
  The button that opens the new worksheet named *Data #* for plugging parameters of input quantities ***X***.
- *MC Trials*
  The edit box that serves for setting the number *M* of Monte Carlo trials.
- *New MC Simulation*
  The button that runs evaluation procedure and generates the new report sheet named *Report #* with evaluation results.

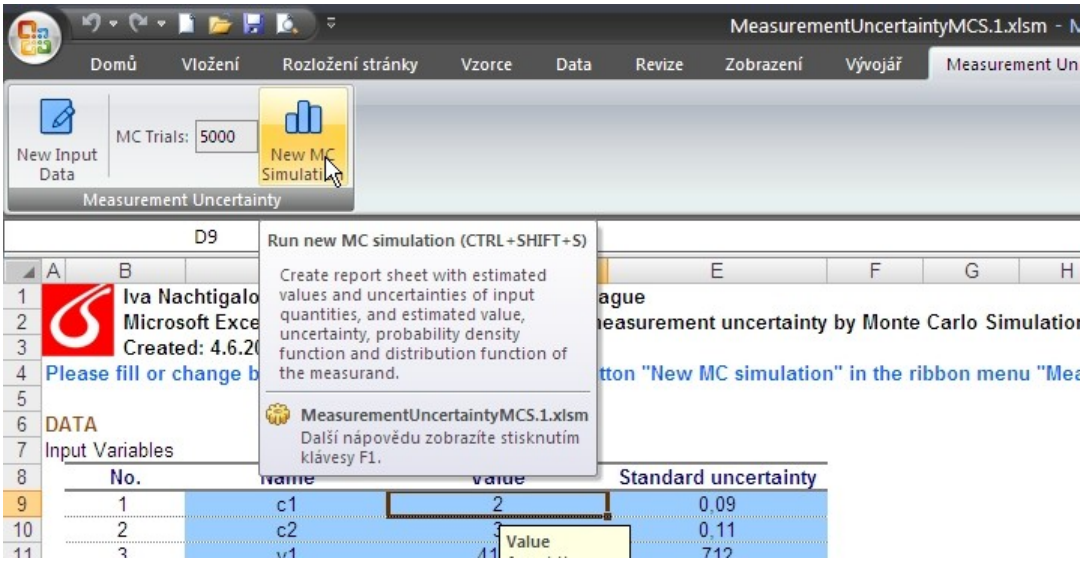


Fig. 2 Tab Measurement Uncertainty and its controls

Note: It is also possible to run the macros associated with controls by shortcuts displaying in the tooltips, or from the list of macros in the dialog of the button *Macros* on the tab *Developer*.

3.2 PLUGGING INPUTS

Inputs for evaluation procedure must be plugged into colored cells of the worksheet that is generated by the button *New Input Data*.

Note: This worksheet is protected against insertion data of other data types than required ones. It is possible to insert only correct data from keyboard or by copying them from the other worksheets (they must be “paste as”, not only “paste”).

Evaluation procedure requires plugging these inputs (Fig. 3):

- *Name*
  The names or descriptions of input quantities of the measurement model.
- *Value*
  The values or (arithmetic) means of input quantities of the measurement model.
- *Standard uncertainty*
  The standard uncertainty of input quantities of the measurement model. Because the model supposes normal distribution of input quantities values the user must always insert standard deviations.

In addition to these required data the user can take optional notes into cells labeled *Notices*.

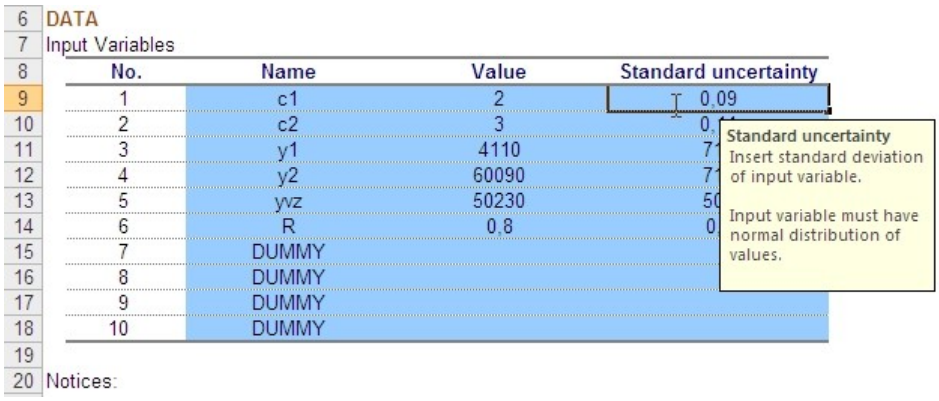


Fig. 3 Worksheet Data # – plugging inputs

3.3 RUNNING EVALUTION AND INTERPRETATION OF RESULTS

Before uncertainty evaluation the user must

- Set number *M* of Monte Carlo trials in the edit box *MC Trials* (minimally 1 000 and maximally 100 000 trials).
- Activate some worksheet with input data.

After these steps the user can start the evaluation procedure by the button *New MC Simulation*. Firstly random samplings of input quantities are generated by Monte Carlo method, then procedure is interrupted and the user is asked to enter the formula *f* of the measurement model in the dialog (Fig. 4).

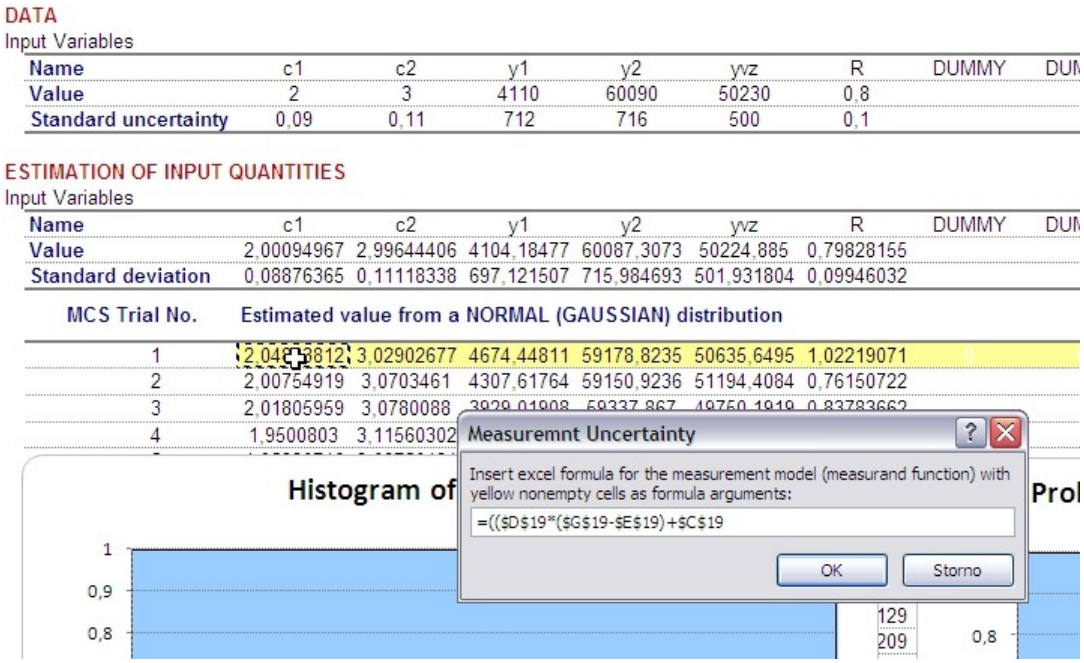


Fig. 4 Worksheet Report # – entering formula of the measurement model

Note: The formula of the measurement model must be Excel formula, i.e. must starts by sign = and can contain only numbers, basic mathematical operators, rounded brackets and cell addresses. Specifically, formula must contain only addresses of yellow-colored nonempty cells (first random samplings of input quantities).

DATA

Input Variables

Name	c1	c2	y1	y2	yvz	R	DUMMY	DUMMY	DUMMY	DUMMY
Value	2	3	4110	60090	50230	0,8				
Standard uncertainty	0,09	0,11	712	716	500	0,1				

ESTIMATION OF INPUT QUANTITIES

Input Variables

Name	c1	c2	y1	y2	yvz	R	DUMMY	DUMMY	DUMMY	DUMMY
Value	1,9983953	3,0025173	4109,2821	60094,572	50235,126	0,7995501				
Standard deviation	0,0896803	0,1103256	723,80514	721,12362	504,64228	0,0992928				

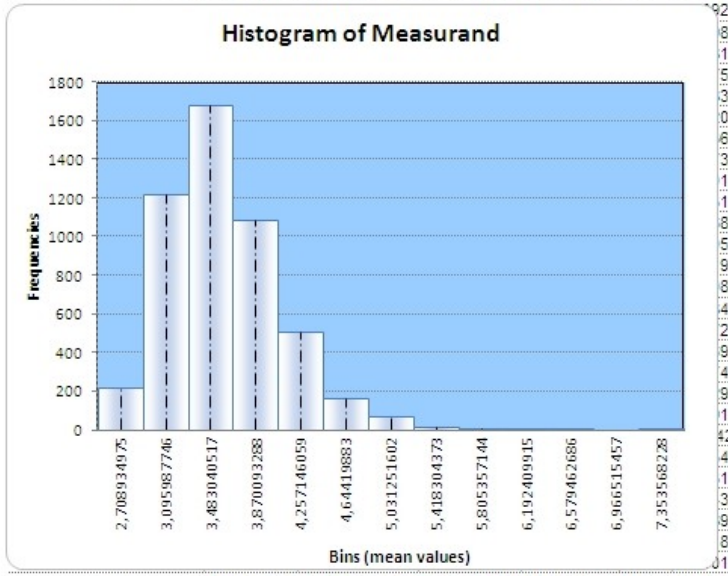
MCS Trial No.	Estimated value from a NORMAL (GAUSSIAN) distribution					
1	1,936662	2,8257393	3567,8707	59255,916	50261,369	0,6688014
2	1,8359096	2,932229	5416,8662	60060,734	48892,699	0,6238996
3	2,0630043	3,0020198	3777,9077	60326,802	50080,454	0,8106518
4	2,0503241	3,028812	5239,6759	61184,113	49875,579	0,7901918
32	2,0417656	2,8982238	3402,258	60051,944	49788,642	0,633618
33	2,1477977	2,9640826	3432,1523	59691,995	49462,184	0,8568444
34	1,9010766	3,0317621	4071,235	59905,53	50794,025	0,8579938
35	2,0657143	3,0279206	4536,4352	60342,061	50033,364	0,9173281
36	2,0580973	3,1746968	5067,0151	60379,149	50425,334	0,752766

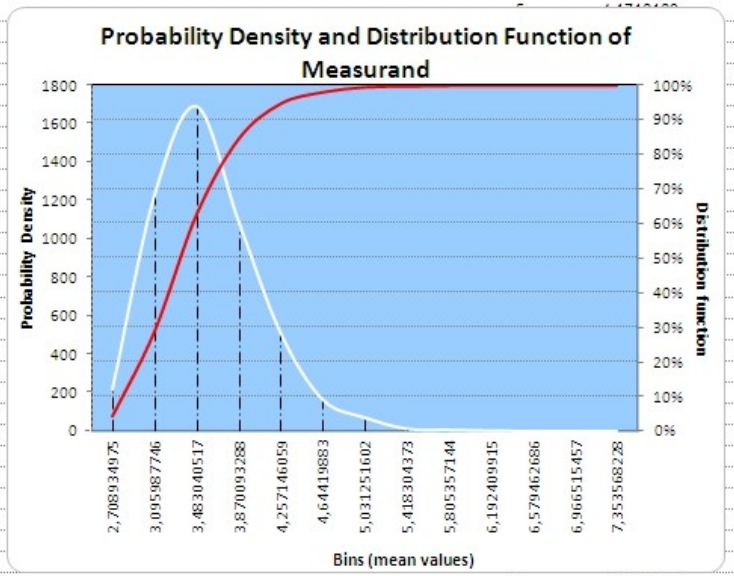
ESTIMATION OF MEASURAND

Measurand

Formula	=(c2*(yvz-y1)+c1*(y2-yvz))/(R*(y2-y1))
Value	3,59228
Standard deviaton	0,491823

No.	Simulated value
1	4,0103659
2	4,340706
3	3,4933312
4	3,5827022
32	4,3291982
33	3,2860764
34	3,318494
35	3,107042
36	3,9504399





ESTIMATED MEASURAN

Parameters

Num of values	
Min	2,5
Max	7,5
Num of bins	
Bins with	0,3

Bins, frequencies and dist

No.	Lov
1	2,5
2	2,9
3	3,2
4	3
5	4,0
6	4,4
7	4,8
8	5,2
9	5,6
10	5,9
11	6
12	6,7
13	7,1

Fig. 5 Worksheet Report # - section DATA with used input data, sections ESTIMATION OF INPUT QUANTITES and ESTIMATON OF MEASURAND with simulated means, standard deviations and estimates in each simulation trial, approximating histogram and graphs of PDF and DF of measurand

After entering the formula the evaluation procedure continues and the user is informed about its state in the Excel status bar.

The evaluation results are saved into the new worksheet: all inputs (including the measurement model) are recapitulated in the section labeled DATA, outputs are in the other sections. Their meaning is as follows (Fig. 5):

- *ESTIMATION OF INPUT QUANTITIES*
  The table of input quantities with their names, simulated means, simulated standard deviations and random sampling values simulated in each Monte Carlo trial.
- *ESTIMATION OF MEASURAND*
  The table of measurand with its function (i.e. formula of the measurement model), simulated mean, simulated standard deviation and values simulated in each Monte Carlo trial.
- *HISTOGRAM OF MEASURAND*
  Histogram of measurand that was plotted from data saved in the table *Bins, Frequencies and Distribution function* of the section *ESTIMATED MEASURAND HISTOGRAM, PROBABILITY DENSITY FUNCTION AND DISTRIBUTION FUNCTION*.
- *GRAPH OF PROBABILITY DENSITY FUNCTION AND DISTRIBUTION FUNCTION OF MEASURAND*
  Graph of measurand functions that was plotted from data saved in the table *Bins, Frequencies and Distribution function* of the section *ESTIMATED MEASURAND HISTOGRAM, PROBABILITY DENSITY FUNCTION AND DISTRIBUTION FUNCTION.*

3.4 REPORT MESSAGES

Report worksheets confirm successful uncertainty evaluation, if not warn the user to possible errors. Messages are always displayed in the row no. 4 under the sheet identification header and they differ in color (Fig. 6)

Fig. 6 Report message confirming successful analysis (on the left) and warning to possible error (on the right)

3.5 PRINTING

If it is necessary worksheets with input data and results of uncertainty evaluation can be printed. Print preview or printing is invoked using standard MS Excel commands.

Note: Print parameters are set to procedure inputs and outputs always include sheet identification header of the current sheet and cover just one A4 page. However after connecting the user's printer, the page margins can be automatically changed and printed data can be distributed to multiple pages. Then user must adjust print parameters manually.

3.6 SAVING

Each workbook based on the template contain VBA codes, which check inputs, generate random samplings of input quantities and evaluation results by the Monte Carlo method, plot graphs and

provide tasks associated with the controls on the Ribbon tab. In order for workbooks to remain fully functional they must be saved as macro-enabled ones (with extension XLSM).

Note: If a workbook is saved as a standard one without enabled macros (with extension XLSX) it irreversibly (!) loses all macros and thus its functionality.

Note: Report worksheets with evaluation results can slow opening and saving a workbook due to a large amount of data and also significantly increase size of a file. It is therefore preferable to work with multiple workbooks in the case of making more analyses.

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