Adaptive Thresholding Using Kriging

Slavíková, P., Mudrová, M.
Department of Computing and Control Engineering,
Institute of Chemical Technology, Prague
Prague, Czech Republic
{slavikop, mudrovam}@vscht.cz

Moravec H.
Department of Metals and Corrosion Engineering
Institute of Chemical Technology, Prague
Prague, Czech Republic
moravech@vscht.cz

***Abstract*—Kriging is proved and commonly used computational method in geological research based on probability. Presented project transfers this approach to image processing and studies application to nanotubes Results are compared with Otsu method.**

***Keywords*— Kriging; Image Segmentation; Nanotubes; Nanomaterial; Image Thresholding)**

I. Introduction

Segmentation is common and often used method for image data evaluation. This simple and effective approach is applied in science and practical use, as well. Examples of practical segmentation application are document binarization, where printed characters are extracted [1]; scene processing, where a target should be found [2]; map processing, where lines, legends and characters are detected [3]; quality inspection of materials, where defective part should be marked [4]. Wide branch of thresholding application is processing of measurement image data from laser scanning microscope [5], x-ray computed tomography [6] or ultrasonic images [7].

Goal of image segmentation is to find and extract feature or pattern in image data which represents desired image information. Choice of proper threshold is essential for successful segmentation process. At general, segmentation methods are based on exploitation of [8]

1) histogram shape information: These methods achieve thresholding based on the shape properties of histogram. Mentioned algorithms study histogram convex hull [9], peaks and valleys [10], histogram smoothing via autoregressive modeling [11] or histogram shape-modeling [12].

2) measurement space clustering: The gray-level data undergo a two channel clustering analysis. Basic principle of these algorithms is in histogram separation into two channel and finding threshold. Otsu method [13], searching of midpoint peaks [14], fuzzy clustering [15, 16] and mean-square clustering methods at general [17, 18] can be examples of this approach.

3) histogram entropy information: This type of methods uses the entropy distribution in the image. A minimum of cross-entropy between input gray-level image and resulting binary image is presented in [19]. Against this, authors of [20] exploit minimization of thresholded image entropy. Advanced study of image entropy corrected by Shannon wavelet function is citing in [21].

4) spatial information: These algorithms find threshold based on similarity or feature quality between original and binarized image, like edge matching [22], texture [23] or stability of segmented objects [24].

5) local characteristics: In this class of algorithms, a threshold is calculated for each pixel and a threshold value reflects local neighborhood. That means, local threshold $T(i, j)$ is function of coordinates (i, j) at each pixel and one image can be segmented by set of thresholding values instead of one. These methods are based on evaluation of local statistics like range [25], local variance [26] and covariance (kriging) [6] or surface-fitting parameters [27].

This paper presents an adaptive thresholding using kriging interpolation method. Kriging is a geostatistical interpolation developed and used in the mining industry for interpolation of input point data and estimation of a block model. The name „kriging“ was given by G. Matheron [29] in honor of the South African mining engineer D. Krige [30].

The most common kriging techniques are variants of ordinary kriging, known as linear kriging. The more complex kriging techniques (indicator kriging, disjunctive kriging, etc.) are based on nonlinear transformation of grades and are grouped as non-linear kriging methods. In image processing, kriging is used to estimate the value of pixel by minimizing the variance of estimate error based on analysis of semi-variances whose values depend on spacing and orientation of pair-wise pixels.

II. Mathematical Background

A. Ordinary Kriging

Basic principle [31] of kriging is an estimation of a function $z(x)$ based on N sampling points $[x_1, x_2, \ldots, x_N]$. It is supposed that $z(x)$ is a realization of random function $Z(x)$. This function is also supposed to be second order stationary, i. e. both expectation and covariance function exists and does not depend on x.

Then, the interpolation (1) could be performed. It is based on an estimate random function $Z^*(x)$ and it is defined as a linear combination of the $Z(x)$ values on the N sampling points.

$$Z^*(x) = \sum_{i=1}^{N} w_i(x) Z(x_i) \tag{1}$$

In computation order of the estimate random function $Z^*(x)$, it is necessary to evaluate the values of all weight functions $w_i(x)$. The random function $Z^*(x)$ must be a good estimator of $Z(x)$, so two conditions are imposed for $w_i(x)$ evaluation: 1. the expectation of the error between $Z(x)$ and $Z^*(x)$ must be zero, 2. the error between $Z(x)$ and $Z^*(x)$ must be minimized. When the mean value is supposed unknown for every x, the first condition allows us to write (2). The second condition (3) is equivalent to the minimization of the variance of $(Z(x) - Z^*(x))$, for every x.

$$\sum_{i=1}^{N} w_i(x) = 1 \tag{2}$$

$$\begin{aligned} Var\left(Z^*(x) - Z(x)\right) = \sum_i \sum_j w_i(x) w_j(x) \operatorname{cov}(x_i, x_j) - \\ -2 \sum_i w_i(x) \operatorname{cov}(x_i, x) + \operatorname{cov}(x, x) \end{aligned} \tag{3}$$

The ordinary Kriging equations are gained when both conditions are taken into account.

When (3) is minimized with respect to $w_i(x)$ constrained by (2), the ordinary Kriging equations are obtained (4).

$$\begin{bmatrix} C & B \\ B^t & 0 \end{bmatrix} \begin{bmatrix} w \\ \lambda \end{bmatrix} = \begin{bmatrix} A \\ 1 \end{bmatrix} \tag{4}$$

Equation (4) shows a linear system with $N + 1$ equations and $N + 1$ unknown weights $[w_1, w_2, \ldots, w_N]^T$. The general term of matrix **C** is the covariance function $\operatorname{cov}(x_i, x_j)$ and its dimension is $N \times N$. **B** is a $N \times 1$ vector filled by ones and λ is a Lagrange multiplier. **A** is also $N \times 1$ vector and its general term is $\operatorname{cov}(x, x_i)$.

Thus, if covariance function is defined, the problem could be solved. Good selection of covariance function is very essence of successful data approximation.

Ordinary kriging interpolation value is defined by (5).

$$z^*(x_i) = \sum_{i=1}^{N} w_i(x) z(x_i) \tag{5}$$

B. Kriging in 3x3 neighborhood

For clear conversion of geostatistical analysis to image processing, it is necessary to use a kriging filter $\mathbf{C_k}$ as the covariance function **C** in (4). Used kriging filter is 3x3 neighborhood connected to central pixel, see Fig. 1. [32, 33, 34]

1	2	3
4	i	5
6	7	8

Fig. 1. 3x3 neighborhood designation

Consider the center pixel to be filtered. This pixel is denoted by i and its neighboring pixels j, where $j = 1, 2, \ldots, 8$. Covariance function **C** corresponding to formal distance between every each pixel is presented in (6). Let a formal distance between two neighboring pixel is equal to 1.

$$C_k = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{18} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{28} \\ \vdots & \vdots & \ddots & \\ \gamma_{81} & \gamma_{82} & & \gamma_{88} \end{bmatrix} \tag{6}$$

where $\gamma_{11} = 0$, $\gamma_{12} = 1$, $\gamma_{13} = 2$, $\gamma_{14} = 1$, $\gamma_{15} = \sqrt{5}$, $\gamma_{16} = 2$, $\gamma_{17} = \sqrt{5}$, $\gamma_{18} = \sqrt{8}$, etc.

Covariance function **A** (7) consists of formal distances of central pixel i and neighboring pixels j and it is assembled by same procedure as above.

$$A = \left[\gamma_{i1} \quad \gamma_{i2} \quad \cdots \quad \gamma_{i8}\right]^T \tag{7}$$

In case of 3x3 neighborhood, it is obvious that $\gamma_{i1} = \gamma_{i2} = \gamma_{i3} = \gamma_{i4} = \gamma_{i5} = \gamma_{i6} = \gamma_{i7} = \gamma_{i8} = 1$.

This procedure can be used for kriging in a various window sizes but it is necessary to consider that a computational time is quadratic increasing with window size.

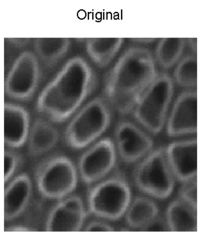
III. Application On Real Image Data

Methods described above were used during processing of real images of nanostructured material. Nanomaterials provide a new way in improving technologies in many fields of science. They find use in industrial applications, medicine, information technologies and others. Development of new materials and processes based on nano-dimensioned appliances is a very topical issue [35, 36]. A new generation of materials with surface parts or whole surface structure in the nanometer scale shows different properties in comparison with standard materials. Images of nanostructures on Titanium Grade 2 are investigated by means of image processing methods including thresholding.

A. Adaptive thresholding using kriging

Preprocessed image was separated to single areas and each section was processed apart. Kriging was applied at selected window of image data by principle described in part II.B. Then window moved pixel by pixel until whole area wasn't thresholded. Resulting matrix of thresholds was treated by median and mean value to gain an overall threshold for each nanostructure section. For comparison with kriging Otsu thresholding method was applied, as well.

A cut-out of nanostructure material image is shown at Fig. 2 and a detail of area No. 685 is represented at Fig. 3.



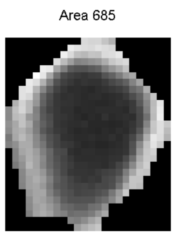


Fig. 2. Cut-out of nanomaterial structure image

Fig. 3. Detail of selected nanotube

IV. Results

Goal of image processing is to separate wall and center of nanotubes for their evaluation. As you can see, both wall and center has several gray-levels. This occurrence is caused by different grown height of nanostructure during preparation and it excludes using global thresholding methods. There are presented threshold values gained by Otsu method and by kriging treated by median and mean value for selected areas in Table 1.

TABLE I. THRESHOLD VALUE FOR SELECTED NANOSTRUCTURE SEGMENTS

Area No.	**Threshold Value**		
	Kriging		***Otsu method***
	Median	***Mean value***	
5	49,5	59,4	84,0
36	57,5	53,8	57,0
112	59,0	55,4	170,0
245	66,0	54,7	176,0
305	67,0	58,5	194,0
428	61,0	60,4	65,5
513	73,0	83,1	55,0
685	45,0	53,1	173,0
756	56,0	55,2	171,0
799	82,0	74,0	66,0

It is obvious, that Otsu method gives unreliable results. More than half of gained results almost rise to maximum image intensity and thresholding process is corrupted see Figs. 4, 5.

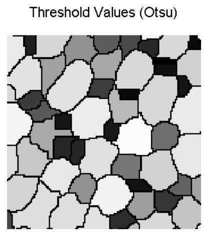
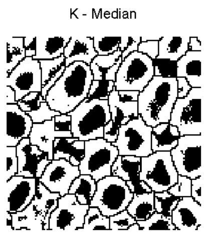


Fig. 4. Binary image of Fig. 2 gained by Otsu method

Fig. 5. Threshold values of Fig. 2 gained by Otsu method

Examples of image results for kriging are presented at Figs. 6 – 9. Thresholding quality is much better in comparison with Otsu thresholding method. Walls and centers of nanotubes are recognizable and separated.



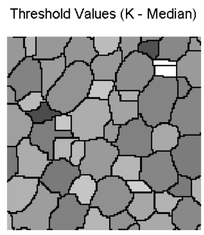
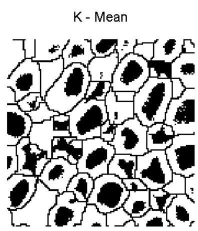


Fig. 6. Binary image of Fig. 2 gained by kriging (median)

Fig. 7. Threshold values of Fig. 2 gained by kriging (median)



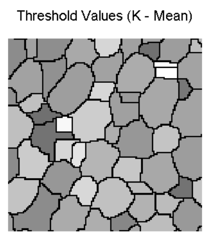


Fig. 8. Binary image of Fig. 2 gained by kriging (mean value)

Fig. 9. Threshold values of Fig. 2 gained by kriging (mean value)

Consequential problem is selection of window size for kriging. Area No. 685 was chosen for testing of modifications in threshold evaluation. According to Fig. 10, threshold values gained by median adjusted kriging are changing rapidly with window size. In comparison, mean value adjusted kriging is less sensitive to window size change.

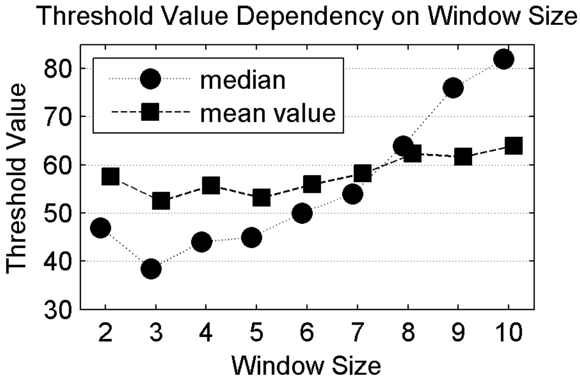


Fig. 10. Threshold Dependency on Window Size for Kriging Adjusted by Mean Value and Median

V. CONCLUSION

Presented results show that a typical geostatistic method kriging is useful for evaluation of nanostructure image data. It is necessary to segment input data and evaluate each area separated to gain a satisfying output.

ACKNOWLEDGMENT

This work has been supported by the Ministry of Education of the Czech Republic (program No. MSM 6046137306 and MSM 6046137302).

REFERENCES

[1] M. Kamel, and A. Zhao, "Extraction of binary character/graphics images from grayscale document images," Graph. Models Image Process, Vol. 55, pp. 203-2017, 1993.

[2] B. Bhanu, "Automatic target recognition: state of art survey," IEEE Trans. Aerosp. Electron. Syst., vol. 22, pp. 364-379, 1986.

[3] O. D. Trier, and A. K. Jain, "Goal-directed evaluation of binarization methods," IEEE Trans. Pattern Anal. Mach. Intel., vol. 17, pp. 1191-1201, 1995.

[4] M. Sezgin, and R. Tasaltin, "A new dichotomization technique to multilevel thresholding devoted to inspection applications," Pattern Recogn. Lett., vol. 21, pp. 151-161, 2000.

[5] H. J. Trussel, "Comments on picture thresholding using iterative selection method," IEEE Trans. Syst. Man Cybern., vol. 9, pp. 311, 1979.

[6] W.Oh, and W. B. Lindquist, "Image thresholding by indicator kriging," IEEE Trans. Pattern Anal. Mach. Intell., vol. 21, pp. 590-602, 1999.

[7] L. U. Wu, M. A. Songde, and L. U. Hanqing, "An effective entropic thresholding for ultrasonic imaging," ICPR '98: Intl. Conf. Patt. Recog., pp. 1522-1524, 1998.

[8] M. Sezgin, and B. Sankur, "Survey over image thresholding techniques and quantitative performance evaluation," J. Electron. Imaging, vol. 13, pp.146-165, January 2004.

[9] R. J. Whatmough, "Automatic threshold selection from a histogram using the exponential hull," Graph. Models Image Process., vol.53, pp. 592-600, 1991.

[10] M. I. Sezan, "A peak detection algorithm and its application to histogram-based image data reduction," Graph. Models Image Process., vol. 29, pp. 47-59, 1985.

[11] J. Cai, and Z.Q. Liu, "A new thresholding algorithm based on all-pole model, ICPR '98, Intl. Conf. Patt. Recog., pp. 34-36, 1998.

[12] M. J. Carlotto, "Histogram analysis using a scal-space approach," IEEE Trans. Pattern Anal. Mach. Intell, vol. 9, pp.121-129, 1997.

[13] N. Otsu, "A threshold selection method from gray-level histograms," IEEE Trans. Syst. Man Cybern., vol. 9, pp. 62-66, 1979.

[14] C. K. Leung, and F. K. Lam, "Performance analysis of a class of iterative image thresholding algoritms," Pattern Recogn., vol. 29, pp. 1532-1530, 1996.

[15] C. V. Jawahar, P. K. Biswas, and A. K. Ray, "Investigation on fuzzy thresholding based on fuzzy clustering," Pattern Recogn., vol. 30, pp. 1605-1613, 1997.

[16] N. Ramesh, J. H. Yoo, and I. K. Sethi, "Thresholding based on histogram approximation," IEEE Proc. Vision Image Signal Process., vol. 142, pp. 271-279, 1995.

[17] T. W. Ridler, and S. Calvard, "Picture thresholding using an iterative selection method," IEEE Trans. Syst. Man Cybern., vol. 8, pp. 630-632, 1978.

[18] H. J. Trussel, "Comments on picture thresholding using iterative selection method," IEEE Trans. Syst. Man Cybern., vol. 9, pp. 311, 1979.

[19] C. H. Li, and C. K. Lee, "Minimun cross-entropy thresholding," Patern. Recogn., vol. 26, pp. 617-652, 1993.

[20] J. N. Kapur, P. K. Sahoo, and A. K. C. Wong, "A new method for gray-level picture thresholdilng using the entropy of the histogram," Graph. Models Image Process., vol. 29, pp. 273-285, 1985.

[21] G. Johannsen, and J. Bille, "A threshold selection method using information measures," ICPR '82: Proc. 6th Intl. Conf. Pattern Recogn., pp. 140-143, 1982.

[22] S. Venkatesh, ans P. L. Rosin, "Dynamic threshold determination by local and global edge evaluation," CVGIP: Graph. Models Image Process., vol. 57, pp. 146-160, 1995.

[23] Y. Liu, and S. N. Srihari, "Document image binarization based on texture analysis," Proc. SPIE, vol. 2181, pp. 254-263, 1994.

[24] A. Pikaz, and A. Averbuch, "Digital image thresholding based on topological stable state," Pattern Recogn., vol. 29, pp. 829-843, 1996.

[25] J. M. White, and G. D. Rohrer, "Image thresholding for optical character recognition and other application requiring characted image extraction," IBM J. Res. Dev., vol. 27, pp. 400-411, 1983.

[26] J. Sauvola and M. Pietasinen, "Adaptive document image binarization," Patern Recogn., vol. 33, pp. 225-236, 2000.

[27] S. D. Yanovitz, and A.M. Bruckstein, "A new method for image segmentation," Comput. Graph. Image Process., vol, 46, pp. 82-95, 1989.

[28] D. G. Krige, "A statistical approach to some basic mine valuation problems on the Witwatersrand," J. Chem., Metal. and Mining Soc. Of South Africa, vol. 52, pp. 119-139, 1951.

[29] G. Matheron, "Principles of geaostatistics," Economic Geology, vol. 58, pp. 1246-1266, 1963.

[30] M. R. Stytz, and R. W. Parrot, "Using kriging for 3D medical imaging," Comp. Med. Imag. and Graph., vol. 17, pp. 421-442, November-December 1993.

[31] G. Host, "Kriging by local polynomials," Comput. Stat. Data Anal., vol. 29, pp. 295-312, 1999.

[32] R. C. Gonzalez, and R. E. Woods, Digital image processing, 3rd ed., New Yersey: Pearson Prentice Hall, 2008.

[33] J. C. Davis, Statistics and Data Analysis, New York: John Wiley & Sons, 2002.

[34] T. Pham, and M. Wagner, "Filtering noisy images using kriging," In Proceedings of Fifth International Symposium on Signal Processing and its Applications, ISSPA '99, Brisbane, Australia, 22-25 August 1999.

[35] Petrová J., Mudrová M., Procházka A., and Fojt J., "Application of Mathematical Morphology on Nanostructure Image Processing," Proceedings of the 18th International Conference on Process Control. Slovak University of Technology in Bratislava, pp. 273-277, 2011.

[36] F. Variola, F. Vertone, L. Richrt, P. Jedrzejowski, J. H. Yi, S. Zalzal, S. Clair, A. Sarkissia, D. F. Perepichka, J. D. Wuest, F. Rosei, and A. Nanci, "Improving biocompatibility of implantable metals by nanoscale

modification of surfaces: an overview of strategies, fabrication methods, and challenges," Small, vol. 5, pp. 996-1006, 2009.