Matlab Toolbox for Data Modeling Using GMM

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Abstract
The presented toolbox contains several functions for data modeling using Gaussian Mixture Model (GMM) in its simplest form, i.e. sum of two Gaussian probability density functions (PDF). The parameters of GMM are estimated by using equation system derived by method of moments. GMM should model a signal and a noise in wavelet domain or in special cases also in spatial domain.

1 Gaussian mixture model
In this paper, image (signal) is modeled by the GMM in the wavelet domain. The GMM [1] is generally given by a mixture of a certain number of Gaussian PDFs with the variances \( \sigma_k \) and mean values \( \mu_k \)

\[
p(x) = \sum_{k=1}^{K} \alpha_k \mathcal{N}(x; \mu_k, \sigma_k^2),\]

(1)

where \( \alpha_k \) are the proportions of the mixture. The parameters \( \alpha_k \) satisfy the constraint \( \sum_{k=1}^{K} \alpha_k = 1 \). If \( K = 2 \), GMM is given by

\[
p(x) = \alpha \mathcal{N}(x; \mu_1, \sigma_1^2) + (1 - \alpha) \mathcal{N}(x; \mu_2, \sigma_2^2).
\]

(2)

The model given by (2) will be utilized in this paper for image modeling while mean value \( \mu_1 \) and \( \mu_2 \) are equal to zero.

The choice of two Gaussian PDF in GMM is a result of compromise between the solvability of the system of moment equations and the quality of the fit.
1.1 Derived system of equations

Let us consider image $X$ in the wavelet domain. The second central theoretical moment [2] of $X$ is given by

$$m_2(X) = \alpha_X \sigma_{1X}^2 + (1 - \alpha_X)\sigma_{2X}^2$$

(3)

and the fourth moment runs as

$$m_4(X) = 3\alpha_X \sigma_{1X}^4 + 3(1 - \alpha_X)\sigma_{2X}^4,$$

(4)

where $\sigma_{1X}$ denotes the first model variance corresponding to image and $\sigma_{2X}$ represents the second model variance.

As a result, we derived two equations with three unknowns. Since we still have two equations with three unknowns, we exploit the the kurtosis $\kappa_N$ [3] given by

$$\kappa_N = \frac{m_4(X)}{m_2(X)}.$$

(5)

From (3) and (4), we derive

$$\kappa_X = \frac{3\alpha_X \sigma_{1X}^4 + 3(1 - \alpha_X)\sigma_{2X}^4}{\alpha_X^2 \sigma_{1X}^4 + 2\alpha_X \sigma_{1X}^2 \sigma_{2X}^2 (1 - \alpha_X) + (1 - \alpha_X)^2 \sigma_{2X}^4},$$

(6)

where the theoretical moments can be substituted by the sample moments $M_2(X)$ and $M_4(X)$ computed via $M_k(X) = \frac{1}{I} \sum_{i=1}^{I} (X_i - E(X))^k$. The first term after division should be equal to $\kappa_N \approx 3/\alpha_X$ (for $\alpha_X - 1 \rightarrow 0$). We empirically found that only this first term after division can be used for estimation of $\alpha_X$. The variances $\sigma_{1X}$ and $\sigma_{2X}$ are estimated utilizing (3) and (4) in the following manner

$$\sigma_{1X}^2 (3\alpha_X \sigma_{1X}^2 - 6\alpha_X m_2(X)) + \alpha_X m_4(X) - m_4(X) + 3m_2^2(X) = 0$$

(7)

$$\sigma_{2X}^2 = \frac{m_2(X) - \alpha_X \sigma_{1X}^2}{1 - \alpha_X}.$$  

(8)

The process of model parameters estimation may be simplified using the following equality $\sigma_{1X} = x_{0.999}/3$, where $x_{0.999}$ denotes the 99.9th percentile. The parameters estimation highly depends on the estimation quality of the sample moments.
2 Conclusion

The derived equation system is implemented in Matlab m-file GMMEstPar.m, whereas the toolbox contains also additional files such as OptHistEval.m (histogram optimization [4]), JefDiv.m (Jeffrey divergence evaluation [5]) and file for algorithm demonstration demoGMM.m. The presented algorithm is not so robust such as an expectation-maximization (EM) algorithm [6], but it is simple. Furthermore, it doesn’t requires all data, but only measured two sample moments (second and fourth).

Acknowledgements

This work has been supported by the research project MSM 6046137306 of the Ministry of Education, Youth and Sports of the Czech Republic.

References


