20. BOUNDARY PROBLEM

Solution of equation: $f(x, y, y', y'') = 0$, $y(x_a) = y_a$, $y(x_b) = y_b$

20.1 Shooting Method

Principle:
1. Estimate of the second (and further) initial conditions: $\widehat{y}'(x_a)$
2. Initial value problem solution to find $\hat{y}(x_b)$
3. The new estimate of the second initial condition:
   $\widehat{y}'(x_a) = \widehat{y}'(x_a) - \alpha\,(\hat{y}(x_b) - y(x_b))$ for a chosen α

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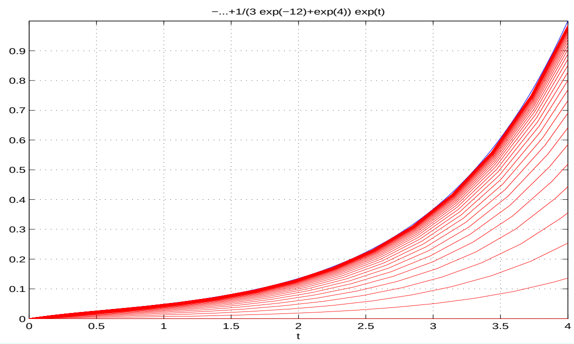
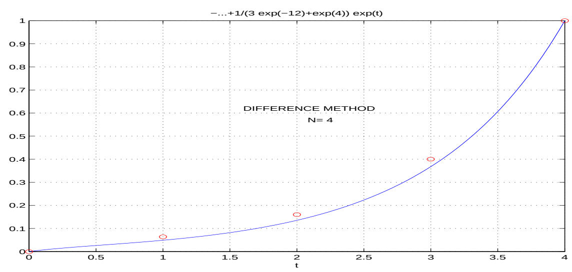
%%% Example 20.1: Ordinary Differential Equations - Boundary Problem
%%% Shooting Method: y''+2y'-3y=0, y(xa)=0; y(xb)=1 for xa=0, xb=4
%%%%%% A. Symbolic Solution %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
  clear; delete(get(0,'children'))
  syms t; xa=0; xb=4;
  ys=dsolve('D2y+2*Dy-3*y=0','y(0)=0','Dy(4)=1');
    pretty(simplify(ys));
    ezplot(ys,[xa xb]); grid on; axis tight
%%%%%% B. Numeric Solution: Shooting Method %%%%%%%%%%%%%
% Equation: y1'=-2*y1+3*y2; y1(xa)=...
%           y2=y1;          y2(xa)=0;
  y1a=input('Volba pocatecni podminky (=0): ');
  y2a=0; y2b=1; alpha=0.01;
  for i=1:30
    [x,y]=ode23('f',[xa xb],[y1a y2a]);
     hold on; plot(x,y(:,2),'r'); axis tight; hold off
    y1a=y1a-alpha*(y(end,2)-y2b); end
```



20.2 Difference Method

Principle:
1. Division of the range $\langle x_a, x_b \rangle$ into N strips defining $x_k = xa + (k-1)\,h$ for $k = 2, 3, \cdots, N$ and $h = (xb - xa)/N$
2. Application of difference approximation of derivatives in equation $f(x_k, y_k, y'_k, y''_k) = 0$ for $k = 2, 3, \cdots, N$
3. Solution of the system of equations: $f(x_k, y_k, 1/h\,(y_{k+1} - yk - 1), 1/h^2\,(y_{k-1} - 2\,y_k + yk + 1)) = 0$
   for $k = 2, 3, \cdots, N$ where $y_1 = ya$ and $y_{N+1} = yb$

```
%%% Example 20.2: Ordinary Differential Equations - Boundary Problem
%%% Difference Method: y''+2y'-3y=0, y(xa)=0; y(xb)=1 for xa=0, xb=4
%%%%%% A. Symbolic Solution %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
  clear; delete(get(0,'children'))
  syms t; xa=0; xb=4;
  ys=dsolve('D2y+2*Dy-3*y=0','y(0)=0','Dy(4)=1');
    pretty(simplify(ys));
    ezplot(ys,[xa xb]); grid on; axis tight
%%%%%% B. Numeric Solution: Difference Method %%%%%%%%%%%
% Equation: y''(k)+2*y(k)+3*y(k); y(xa)=0; y(xb)=1;
%     h=(xb-xa)/N;
%     (1/h^2)(y(k-1)-2y(k)+y(k+1)) + 2 (1/(2h))(y(k+1)-y(k-1)) - 3 y(k)=0
  N=4; h=(xb-xa)/N; x=xa:h:xb;
  ya=0; yb=1;
  A=[(-2/h^2-3)   (1/h^2+1/h)    0
     (1/h^2-1/h) (-2/h^2-3)  (1/h^2+1/h)
          0     (1/h^2-1/h) (-2/h^2-3)  ];
  b= [-(1/h^2-1/h)*ya  0  -(1/h^2+1/h)*yb]';  y=inv(A)*b; y=[ya; y; yb];
  hold on; t=plot(x,y,'or'); axis tight; hold off
```




```
 function yd=f(x,y)
% Function for Ex.20.1
   yd=[-2*y(1)+3*y(2); y(1)];
```

COMMANDS

SYMS
DSOLVE
PRETTY
EZPLOT

ODE23
INV
PLOT

EXAMPLES 20

20.1 Evaluate symbolic solution of a boundary value problem for ordinary differential equations using the shooting method

20.1 Evaluate symbolic solution of a boundary value problem for ordinary differential equations using the finite difference method