WAVELET USE FOR NOISE REJECTION AND SIGNAL MODELLING

Martina MUDROVÁ [1], *Aleš PROCHÁZKA* [1], *Jaroslav FIALA* [2]

[1] *Prague University of Chemical Technology, Department of Computing and Control Engineering*
Technická 1905, 166 28 Prague 6, Czech Republic
Phone:+42-2-2435 4198 ⋆ Fax:+42-2-2431 1082 ⋆ E-mail: mudrovam@vscht.cz, prochaz@vscht.cz

[2] *Czech Hydrometeorological Institute, Division of Air Quality Protection*
Na Šabatce 17, 143 06 Prague 4, Czech Republic
Phone:+42-2-401 9801 ⋆ Fax:+42-2-4403 2468 ⋆ E-mail: fialaj@chmi.cz

ABSTRACT

Wavelet functions form an essential tool of modern signal analysis and modelling in various engineering, biomedical and environmental systems. The paper is devoted to their use for time-scale signal analysis and noise rejection. The main part of the paper presents basic principles of signal decomposition in connection with fundamental signal components approximation. Associated methods are verified for simulated signals at first and then used for real signal analysis. Results of such a signal preprocessing are applied for linear and nonlinear modelling and prediction of air pollution inside a given region of North Bohemia covered by several measuring stations. The paper emphasizes algorithmic approach to methods of multichannel signal analysis and prediction by wavelet functions resulting in general methods validated in the Matlab environment.

1 INTRODUCTION

Signal analysis is one of essential parts of digital signal processing in various information and control systems often based upon different discrete transforms. The Wavelet transform forms in many cases a very efficient alternative to the short-time Fourier transform enabling signal analysis with varying time-frequency resolution. Its application further includes signal components detection and signal decomposition, data compression and noise rejection both for one-dimensional and two-dimensional signals [11, 12, 20]. The use of wavelet and radial basis functions as transfer functions of neural networks for signal modelling and prediction is presented in various papers as well [3, 6, 8, 17, 22]. Problems closely related to this topic include signal restoration and segmentation [4, 15], statistical signal processing [10] and efficient optimization methods design [5, 16].

The theory of wavelet transform is studied in many papers and books published recently [2, 12, 18, 19, 21]. Relations between wavelet transform and short-time Fourier transform are discussed in [7]. The analysis and the construction of wavelet functions both in the analytical and in the recurrent forms with description of their properties are important results of these studies.

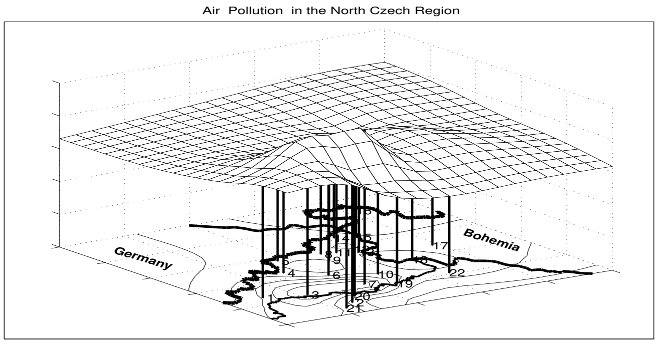

Fig. 1: Air pollution in North Bohemia extrapolated from simultaneous measurements of 22 stations at a selected time

The paper is devoted to basic principles of signal analysis and decomposition by wavelet transform in connection with rejection of noise components forming an additive part of a given sequence. Results of this study are applied for a given multichannel signal interpolation representing air pollution in North Bohemia given in Fig. 1 and its prediction providing comparison with some other related methods [1, 9, 14].

2 SIGNAL ANALYSIS AND WAVELET DECOMPOSITION

Wavelet transform (WT) provides an alternative to the short-time Fourier transform (STFT) for non-stationary signal analysis. Both STFT and WT result in signal decomposition into two-dimensional function of time and frequency respectively scale that is closely related to frequency. The basic difference between these two transforms is in the construction of the window function which has a constant length in case of the STFT (including rectangular, Blackman and other window functions) while in case of the WT wide windows are applied for low frequencies and short windows for high frequencies to ensure constant time-frequency resolution. Local and global

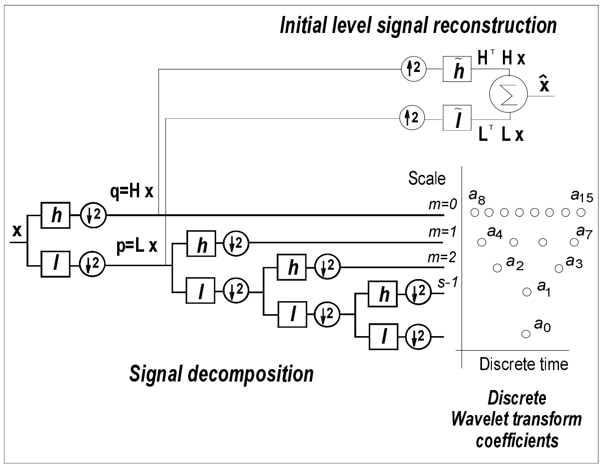

Fig. 2: A pyramidal filter bank structure used to evaluate wavelet transform coefficients for a given sequence $\{x(n)\}_{n=0}^{N-1}$, complementary low pass filter l and high pass filter h and the final coefficients presentation for $s = 4$ and $N = 2^s = 16$

signal analysis can be combined in this way.

In case of continuous (non-stationary) signal $x(t)$ the continuous WT coefficients are given by relation

$$C(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) h(\frac{t-b}{a}) dt \tag{1}$$

where $h(t)$ is the basic (mother) wavelet function and values a and b stand for its translation and dilation respectively. The discrete WT assumes the evaluation of wavelet coefficients for discrete parameters of dilation $a = 2^j$ and translation $b = k\ 2^j$ only. There are strict criteria for the choice of the basic wavelet function given by a requirement of the transforms reversibility [7, 11]. The most frequently used wavelets include Daubechies, Shannon, Morlet, Gaussian and harmonic functions [7, 12].

A basic effficient way to implement this scheme is given by the Mallat's pyramidal structure of WT transform coefficients evaluation for a given column vector $\{x(n)\}_{n=0}^{N-1}$ given in Fig. 2. Using the signal processing point of view this algorithm assumes the half band low-pass scaling sequence $\{l(n)\}_{n=0}^{L-1}$ use together with the corresponding wavelet sequence $\{h(n)\}_{n=0}^{L-1}$ to be convolved with the analyzed signal $\{x(n)\}$ and subsampled by two. These sequences can be derived from functionally given scaling and wavelet functions or defined through the discrete dilation equations [12]. Introducing decomposition matrices

$$\mathbf{L}_{N/2,N} = \begin{bmatrix} l(1) & l(0) & 0 & 0 & \cdots \\ l(3) & l(2) & l(1) & l(0) & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & l(L-1) & \cdots & l(0) \end{bmatrix} \tag{2}$$

$$\mathbf{H}_{N/2,N} = \begin{bmatrix} h(1) & h(0) & 0 & 0 & \cdots \\ h(3) & h(2) & h(1) & h(0) & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & h(L-1) & \cdots & h(0) \end{bmatrix} \tag{3}$$

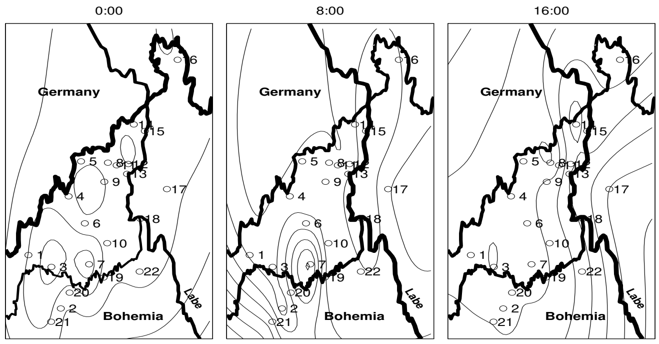

Fig. 3: One day evolution of air pollution in North Bohemia

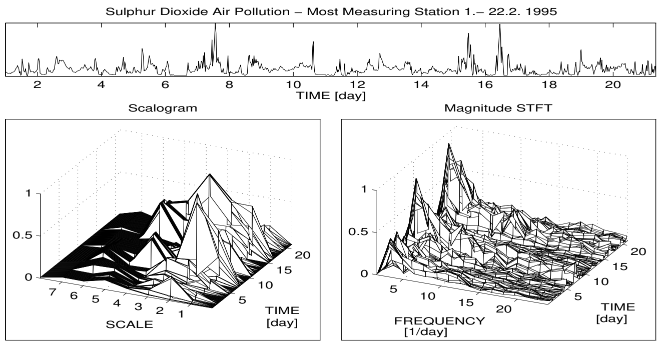

Fig. 4: Spectrogram and scalogram of a given data segment representing air pollution measured in Most during a three weeks period

it is possible to define the separation process for resulting signals $\mathbf{p}$ and $\mathbf{q}$ in the form

$$[\mathbf{p};\mathbf{q}] = \mathbf{L}\ \mathbf{x} + \mathbf{H}\ \mathbf{x} \tag{4}$$

Introducing further transposed matrices $\mathbf{L}_{N,N/2}^T$ and $\mathbf{H}_{N,N/2}^T$ for signal reconstruction according to Fig. 2 it is possible to evaluate resulting signal

$$\mathbf{z} = (\mathbf{L}^T * \mathbf{L} + \mathbf{H}^T * \mathbf{H}) * \mathbf{x} = \mathbf{T}^T * \mathbf{T} * \mathbf{x} \tag{5}$$

The demand for signal perfect reconstruction results in necessary and sufficient condition for matrix $\mathbf{E} = \mathbf{T}^T\mathbf{T}$ to be the unit one. Matrix $\mathbf{T}$ defined by original matrices $\mathbf{L}$ and $\mathbf{H}$ must be orthonormal in this case.

Various wavelet and scaling sequences can be used for signal analysis defined above. The most common choice include Daubechies wavelets [12] even though their frequency characteristics stand for approximation of band-pass filters only. On the other hand harmonic wavelets owing to their spectral properties can have broader application in many engineering problems.

3 REAL SYSTEM DESCRIPTION

The region of North Bohemia represents a very industrial area with problems of air pollution which is monitored by several measuring stations to enable extrapolation and

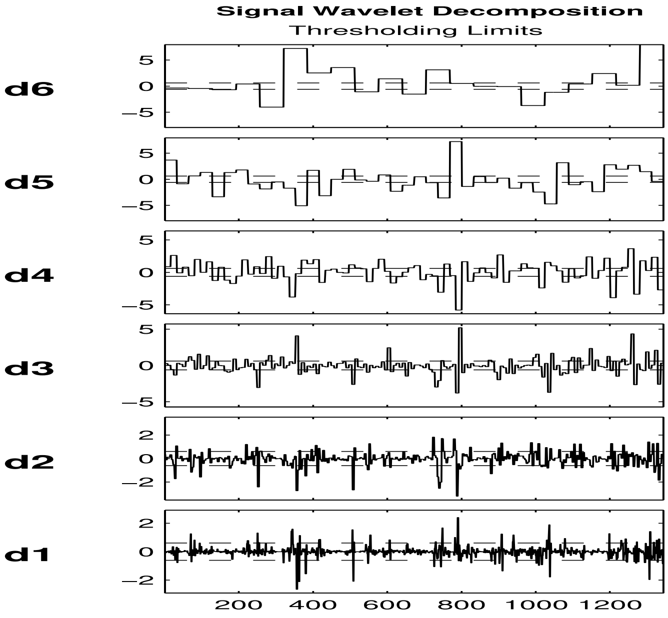

Fig. 5: Wavelet decomposition of a given data segment

prediction of selected pollutants. As some measuring segments can be corrupted it is necessary to interpolate missing values at first. Autoregressive (AR) method introduced in [4] has been applied in this case. The map with the evolution of air pollution during a chosen day is presented in Fig. 3. Analysis of a selected time series is given in Fig. 4 comparing results obtained by the STFT and WT. Owing to its definition the Wavelet transform provides better localization of signal peaks in this case.

4 SIGNAL NOISE REJECTION

Signal decomposition by a given wavelet function into separate levels or scales results in signal compression and it allows the following perfect signal reconstruction again. In case that only selected levels are used it is possible to extract signal components only. Using the threshold method introduced by [20] it is further possible to reject noise and to enlarge signal to noise ratio. This approach has been further modified and applied for a given data segment. Fig. 5 presents signal decomposition into separate levels. Wavelet coefficients are then changed using a chosen thresholding level δ to determine new coefficients

$$\overline{C}(j,k) = \begin{cases} \text{sign } C(j,k)\ (\mid C(j,k) \mid -\delta) & \text{if } \mid C(j,k) \mid > \delta \\ 0 & \text{if } \mid C(j,k) \mid \leq \delta \end{cases}$$

Properly chosen wavelet functions and threshold levels enable to extract very efficiently low pass signal components and to enlarge signal to noise ratio. Fig. 6 provides selected results for such a signal preprocessing which is very useful for further signal modelling and prediction.

5 SIGNAL PREDICTION

Signal prediction has been studied for separate signals at first to compare results achieved by linear autoregressive

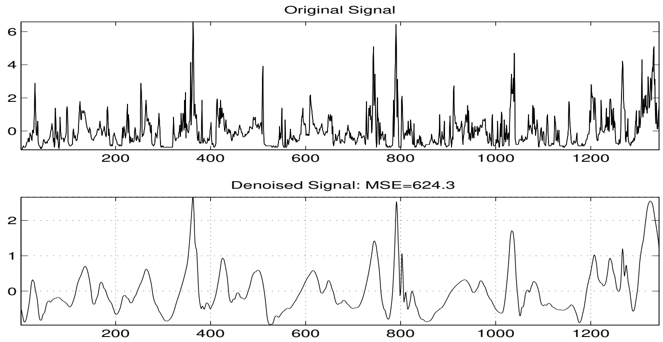

Fig. 6: Wavelet denoising applied for real signal

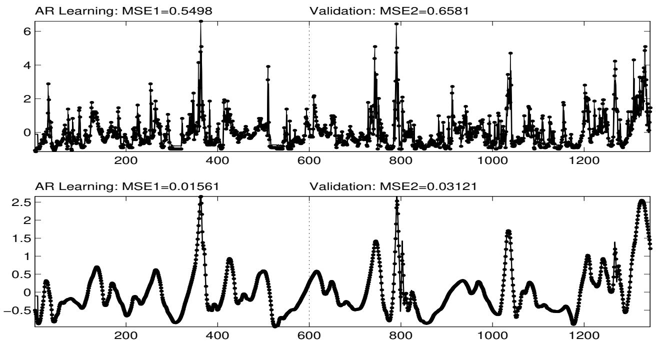

Fig. 7: The prediction of a real signal with its sampling period $T_s = 3$ hours long for the original and denoised signal using the autoregressive model of the 7th order

models [13] and nonlinear models based upon two layers $R - S1 - 1$ neural networks with different transfer functions $F1$ including both sigmoidal and wavelet transfer functions. Output of such a model for any input vector $\mathbf{P}_{R,1}$ can be evaluated by relation

$$\mathbf{Y} = \mathbf{W2}{*}F1\ (\mathbf{W1} * \mathbf{P} + \mathbf{B1}) + \mathbf{B2} \qquad (6)$$

where matrices $\mathbf{W1}_{S1,R}, \mathbf{W2}_{1,S1}$ stand for neural network coefficients and vectors $\mathbf{B1}_{S1,1}, \mathbf{B2}_{1,1}$ represent biases. Selected results of AR modelling are presented in Fig. 7 comparing that achieved both for original signals and denoised signals using wavelet signal decomposition described above. The mean square errors both in the learning and validation parts are summarized in Tab. 1 for various models and original and denoised signals. Experiments resulted in recommendation for signal signal denoising in the preprocessing stage enabling to use autoregressive models with similar prediction errors as selected nonlinear structures.

6 CONCLUSION

The contribution describes selected properties of the wavelet transform compared with that of the short-time Fourier transform. The main part of the paper presents

- the ability of the WT to decompose a given signal
- wavelet coefficients modification for noise rejection
- the following signal reconstruction

TABLE 1: MEAN SQUARE ERROR OF THREE HOURS AIR POLLUTION PREDICTION AT THE MOST STATION

Type	*Original Signal*		*Denoised Signal*	
	Learning	*Valid.*	*Learning*	*Valid.*
AR Model	0.5498	0.6581	0.0156	0.0312
Tansig NN	0.4970	0.7728	0.0096	0.0606
Wavelet NN	0.5412	0.6598	0.0132	0.0328

Methods and algorithm presented in the paper are applied for processing of a real signal representing air pollution to interpolate its missing values, to enlarge its signal to noise ratio and to predict its behaviour. Linear and nonlinear methods are compared to find the most efficient method for given real signal modelling. All results and algorithms were obtained using the MATLAB environment. Further problems include missing values restoration, analysis of wavelet denoising methods, neural network transfer functions choice and their coefficients optimization for signal prediction.

Acknowledgement

The paper has been supported by the grant No. 201/94/0130 of the Grant Agency of Czech Republic. All real signals were kindly provided by the Czech Hydrometeorological Institute in Prague.

Reference

[1] P. Arena, S. Baglio, L. Fortuna, and G. Nunnari. Neural Networks to Predict Ozone Pollution in Industrial Areas. In *VIIIth European Signal Processing Conference EUSIPCO-96*. European Association for Signal Processing, 1996.

[2] I. Daubechies. The Wavelet Transform, Time-Frequency Localization and Signal Analysis. *IEEE Trans. Inform. Theory*, 36:961–1005, Sept. 1990.

[3] B. Delyon, A. Juditsky, and A. Benveniste. Accuracy Analysis for Wavelet Approximation. *IEEE Trans. on Neural Networks*, 6(2):332–348, 1995.

[4] W. Etter. Restoration of Discrete-Time Signal Segments by Interpolation Based on the Left-Sided and Right-Sided Autoregressive Parameters. *IEEE Trans. Signal Processing*, 44(5):1124–1135, 1996.

[5] M. T. Hagan and M. B. Menhaj. Training Feedforward Networks with the Marquardt Algorithm. *IEEE Trans. Neural Networks*, 5(6):989–993, November 1994.

[6] S. Haykin. *Neural Networks*. IEEE Press, New York, 1994.

[7] Y. T. Chan. *Wavelet Basics*. Kluwer Academic Publishers, Boston, 1995.

[8] E. S. Chng, S. Chen, and B. Mulgrew. Gradient Radial Basis Function Networks for Nonlinear and Nonstationary Time Series Prediction. *IEEE Trans. on Neural Networks*, 7(1):190–194, 1996.

[9] P. P. Kanjilal. *Adaptive Prediction and Predictive Control*. Peter Peregrinus Ltd., IEE, U.K., 1995.

[10] S. M. Kay. *Fundaments of Statistical Signal Processing*. Prentice Hall, 1993.

[11] M. Misiti, Y. Misiti, G. Oppenheim, and J. M. Poggi. *Wavelet Toolbox*. The MathWorks, Inc., Natick, Massachusetts 01760, March 1996.

[12] D. E. Newland. *An Introduction to Random Vibrations, Spectral and Wavelet Analysis*. Longman Scientific & Technical, Essex, U.K., third edition, 1994.

[13] J. G. Proakis and D. G. Manolakis. *Digital Signal Processing*. Prentice Hall, 1996.

[14] A. Procházka, M. Mudrová, and J. Fiala. Non-Linear Time Series Modelling and Prediction. *Neural Network World*, 6(2):215–221, 1996.

[15] A. Procházka, M. Sláma, and E. Pelikán. Bayesian Estimator Use in Signal Processing. *Neural Network World*, 6(2):209–213, 1996.

[16] A. Procházka and V. Sýs. Application of Genetic Algorithms for Wavelet Networks Signal Modelling. In *VIIth European Signal Processing Conference EUSIPCO-94*, pages II/1078–II/1081. European Association for Signal Processing, 1994.

[17] A. Procházka and V. Sýs. Time Series Prediction Using Genetically Trained Wavelet Networks. In *Neural Networks for Signal Processing 3 - Proceedings of the 1994 Workshop*, pages 195–203. IEEE, 1994.

[18] O. Rioul and M. Vetterli. Wavelets and Signal Processing. *IEEE SP Magazine*, pages 14–38, October 1991.

[19] G. Strang. Wavelets and Dilation Equations: A brief introduction. *SIAM Review*, 31(4):614–627, December 1989.

[20] G. Strang and T. Nguyen. *Wavelets and Filter Banks*. Wellesley-Cambridge Press, 1996.

[21] M. Vetterli. Wavelets and Filter Banks: Theory and Design. *IEEE Trans. Signal Processing*, 40(9):2207–2232, September 1992.

[22] B. A. Whitehead and T. D. Choate. Cooperative-Competitive Genetic Evolution of Radial Basis Functions Centers and Widths for Time Series Prediction. *IEEE Trans. Neural Networks*, 7(4):869–880, July 1996.