SIGNAL SEGMENTATION USING TIME-SCALE SIGNAL ANALYSIS

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ABSTRACT

Change-point detection is a fundamental problem in many areas of signal segmentation, feature extraction and classification. There are various statistical methods that can be applied to estimate boundaries of signal segments. The paper presents another method based upon the wavelet transform and signal decomposition used to detect signal irregularities. This approach can be very efficient if the initial wavelet function is carefully chosen. Basic methods presented in the paper are verified for simulated sequences at first and then used for biomedical signal segmentation. Resulting algorithms are written in the MATLAB environment.

1 INTRODUCTION

Signal segmentation is one of the fundamental problems of digital signal processing in various information, prediction and control systems. To detect signal change-points it is possible to apply the Wavelet transform (WT) in many cases as a very efficient alternative to the short-time Fourier transform (STFT). The main advantage of the WT is in its ability to analyse signals with varying time-frequency resolution while the STFT uses windows of constant length, thereby limiting its time resolution. The applications of Wavelet transforms include signal components detection and signal decomposition, data compression and noise rejection [1, 6, 11, 12, 13, 16]. Problems closely related to this topic include signal restoration, classification, prediction [4, 14, 15], statistical signal processing [2, 5, 8] and motion estimation [9].

The theory of Wavelet transforms is studied in many papers and books published recently [3, 7, 12, 17]. The analysis and the construction of wavelet functions both in the analytical and in the recurrent forms with description of their properties are important results of these studies.

The paper is devoted to basic principles of signal analysis and decomposition by Wavelet transform in connection with signal segmentation. Results of this study are applied to the analysis of a given biomedical signal representing an EEG observation.

2 WAVELET DECOMPOSITION

The Wavelet transforms provide an alternative to the short-time Fourier transform for non-stationary signal analysis. Both the STFT and WT result in signal decomposition into a two-dimensional function of time and frequency or scale closely related to frequency. The basic difference between these two transforms is in the construction of the window function which has a constant length in the case of the STFT (including rectangular, Blackman and other window functions) while in the case of the WT wide windows are applied for low frequencies and short windows for high frequencies to ensure constant time-frequency resolution. Local and global signal analysis can be combined in this way.

In case of a continuous (non-stationary) signal $x(t)$ it is possible to find its expansion by the (orthogonal) set of functions derived from the initial (mother) wavelet function $W(t)$ dilated and translated by coefficients a and b respectively forming functions

$$W_{a,b}(t) = \frac{1}{\sqrt{a}}\, W(\frac{t-b}{a}) \tag{1}$$

The discrete WT assumes the evaluation of wavelet coefficients for discrete parameters of dilation $a = 2^{-j}$ and translation $b = k\ 2^{-j}$ only. The given signal can then be expanded by their combination in the form

$$x(t) = a_0 + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} c_{j,k}\ W(2^j\ t - k) \tag{2}$$

for $t \in \langle 0, 1)$. In the case of orthogonal wavelet functions it is possible to find expansion coefficients in the form

$$a_0 = \int_0^1 x(t)\ dt \tag{3}$$

$$c_{j,k} = a_{2^j+k} = 2^j \int_0^1 x(t)\ W(2^j\ t - k)\ dt \tag{4}$$

There are strict criteria for the choice of the basic wavelet function given by the requirement of the transform reversibility [7, 11]. The most frequently used wavelets include Daubechies, Shannon, Morlet, Gaussian and harmonic functions [7, 12]. From the signal

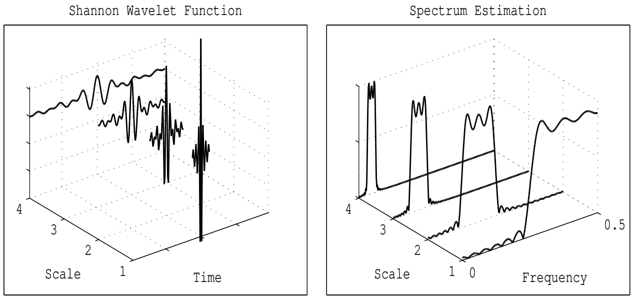

Figure 1: Analysis of the set of Shannon wavelet functions derived from the initial function in the form $W(t) = \sin(\pi\ t/2)/(\pi\ t/2)\ \cos(3\ \pi\ t/2)$ presenting its dilation and the corresponding spectrum compression

processing point of view it is possible to consider wavelet functions as filters with their dilation in the time domain resulting in spectrum compression in the frequency domain. An example of this property is given in Fig. 1 for Shannon wavelet functions.

An efficient way to implement this scheme which is often used is given by the Mallat's pyramidal structure of WT transform coefficients evaluation for a given vector $\{x(n)\}_{n=0}^{N-1}$. The resulting wavelet tree of signal decomposition is given in Fig. 2. From the signal processing point of view this algorithm assumes the use of the half band low-pass scaling sequence $\{l(n)\}_{n=0}^{L-1}$ together with the corresponding wavelet sequence $\{h(n)\}_{n=0}^{L-1}$. These are convolved with the analyzed signal $\{x(n)\}$ and the result of this convolution is subsequently subsampled by two. These sequences can be derived from analytical scaling and wavelet functions or defined through the discrete dilation equations [12]. The solution of a very well known Daubechies dilation equation

$$\Phi(t) = l_0\ \Phi(2\ t) + l_1\ \Phi(2\ t-1) + l_2\ \Phi(2\ t-2) + l_3\ \Phi(2\ t-3) \tag{5}$$

can be achieved by an iterative process in the form

$$\begin{aligned} \Phi_j(t) &= l_0\ \Phi_{j-1}(2\ t) + l_1\ \Phi_{j-1}(2\ t-1) + && (6)\\ &+ l_2\ \Phi_{j-1}(2\ t-2) + l_3\ \Phi_{j-1}(2\ t-3) && (7) \end{aligned}$$

for $j = 1, 2, \cdots$, and a given initial function $\Phi_0(t)$. The wavelet function is then defined by the equation

$$W(t) = -l_3\ \Phi(2\ t) + l_2\ \Phi(2\ t-1) - l_1\ \Phi(2\ t-2) + l_0\ \Phi(2\ t-3) \tag{8}$$

Another approach is based upon the convolution of dilation equation coefficients [10, 11] defining a filter with the initial impulse and upsampling in the successive steps according to Fig. 3. Resulting wavelet and scaling sequences can be used for signal analysis as defined above. The most common choice of wavelets include Daubechies wavelets of various orders even though their frequency characteristics provide only an approximation to band-pass filters. Harmonic wavelets [12] represent another choice that owing to spectral properties enable a wide range of applications in many engineering problems.

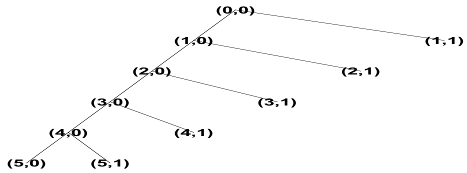

Figure 2: Wavelet tree with indices standing for the depth of signal decomposition

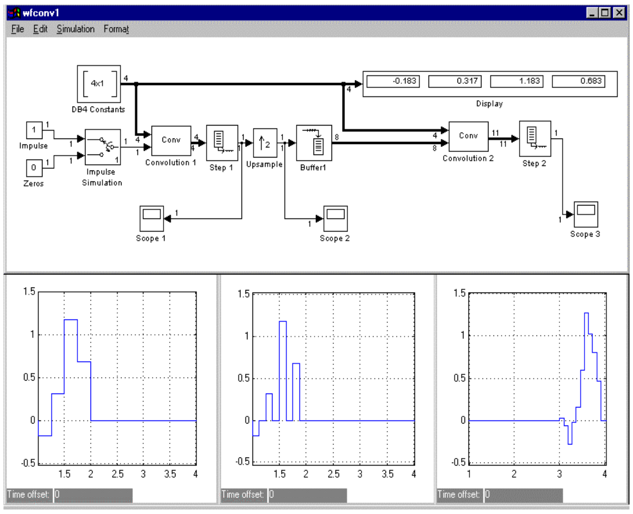

Figure 3: Evaluation of the 4th order Daubechies scaling function using the convolution and upsampling process

3 WAVELET TRANSFORM AND SIGNAL ANALYSIS

Let us study a simulated time series $\{x(n)\}_{n=0}^{N-1}$ formed by two sinusoids with different frequencies and an additive noise according to Fig. 4. The change-point can be detected very precisely after the signal decomposition by the Wavelet transform while the short-time Fourier transform has its time resolution restricted by the chosen window length. Similar results can be achieved for other signals even in the case that the change-point is not so obvious including signals with the change in their derivatives only. The same information can be derived from the analysis of the scalogram. If the initial wavelet function is correctly chosen it is possible to use this method of signal decomposition for signal segmentation as well.

Result of such a process applied for a given part of the EEG signal is presented in Fig. 5. Analysis of a properly chosen level of signal decomposition allows the signal segmentation and classification described in the following section.

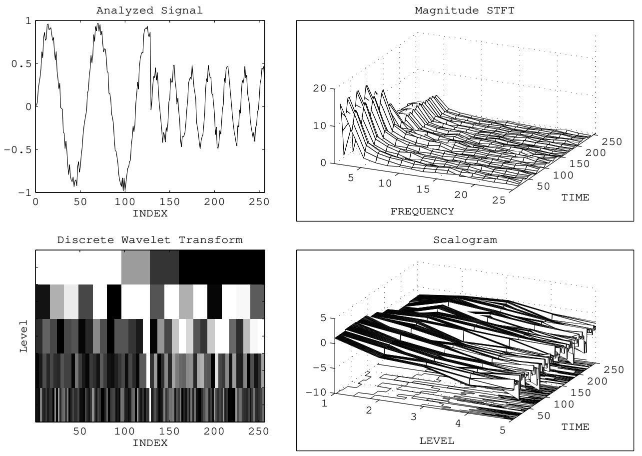

Figure 4: Signal analysis of a given simulated sequence

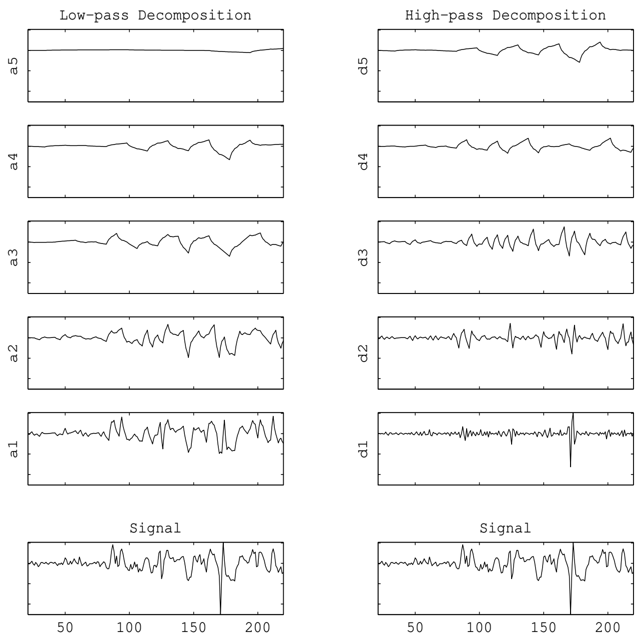

Figure 5: The real signal decomposition by the 4th order Daubechies wavelet functions

4 SIGNAL SEGMENTATION

Wavelet signal decomposition and reconstruction can be used very efficiently both for signal change-points detection and for noise cancelling in many applications. In the case of noise rejection we can eliminate certain levels when the signal is reconstructed. Alternatively one or more of the levels could be modified in some way to eliminate the appropriate features from the reconstructed signal. For example using the threshold method introduced in [16], it is possible to reject the noise and to enlarge the signal-to-noise ratio. Wavelet coefficients are modified using a chosen thresholding level δ to determine new coefficients

$$\overline{C}(j,k) = \begin{cases} \text{sign } C(j,k)\ (\mid C(j,k) \mid - \delta) & \text{if } \mid C(j,k) \mid > \delta \\ 0 & \text{if } \mid C(j,k) \mid \leq \delta \end{cases}$$

In the case of signal segmentation a similar method can be used to find signal indices indicating signal irregularities. An example of such a method applied to an EEG segment analysis is presented in Fig. 6. The squared signal wavelet decomposition for a selected level is compared with a chosen threshold value to find the signal change-points. A scalogram of the chosen signal is given in Fig. 6(C) as a two dimensional scheme while Fig. 7 presents scalogram function in three dimensions. More detailed view of the signal decomposition is important to allow the proper selection of the Wavelet transform level used for signal segmentation to detect important signal features.

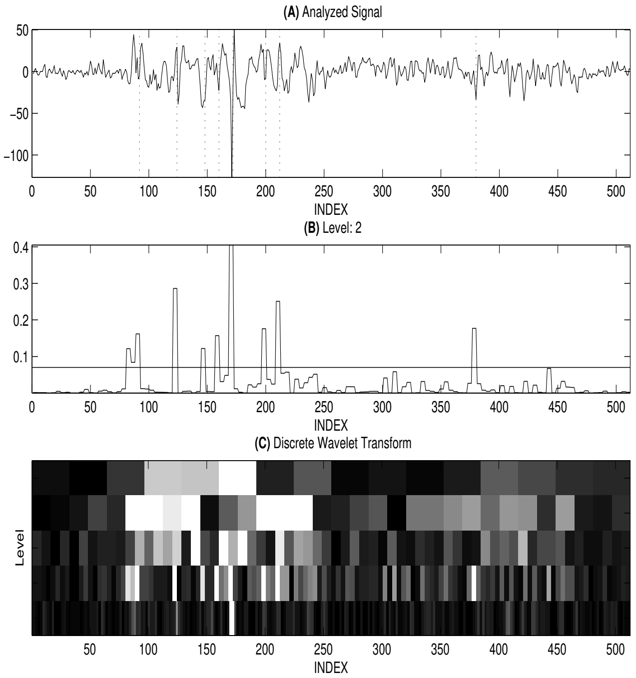

Figure 6: EEG signal segmentation **A:** Given signal with its nine segments defined by the wavelet change-point detection method, **B:** Squared values of the second level of signal wavelet decomposition and a chosen threshold line, **C:** Two dimensional plot standing for five levels of wavelet decomposition coefficients

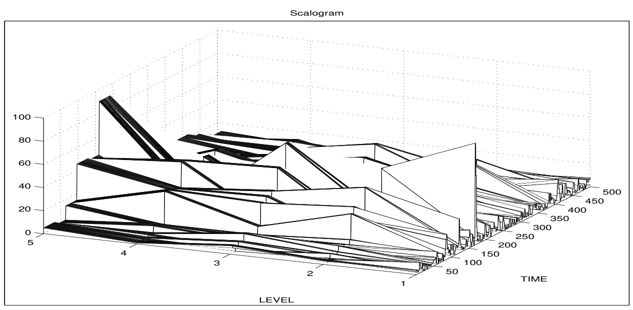

Figure 7: Three dimensional plot representing a scalogram of the analysed EEG signal

Table 1: Frequency components of five classes resulting from classification of EEG signal segments resulting from wavelet change-point detection using competitive neural networks

Class/Segments	*Average Frequency Components*	
	Theta ($4-8\ Hz$)	*Alpha* ($8-13\ Hz$)
A: 1	46.1	33.3
B: 8,9	138.1	46.9
C: 6	212.1	122.9
D: 5,7	87.5	57.3
E: 2,3,4	122.0	53.3

Classification of signal segments into a chosen number of classes is usually based upon features characteristic for signal components. In the case of the EEG signal it is possible to use average frequency contents in its given frequency bands. Results of the subsequent classification based upon competitive artificial neural networks is summarized in Table 1 for the EEG signal studied above.

5 CONCLUSION

This paper presents a study of the use of wavelet decomposition for the detection of signal change-points and for signal segmentation. Algorithms presented in the paper provide similar results as commonly used statistical methods but they provide a more flexible approach owing to the possibility of the choice of various wavelet functions.

The goal of signal segmentation is in most cases in the subsequent signal classification. For the proper choice of segment features it is possible to apply Wavelet transforms again.

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