SIGNAL ANALYSIS AND PREDICTION IN INFORMATION AND CONTROL SYSTEMS

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Abstract: Signal and image analysis represent fundamental problems in many engineering, biomedical, econometric and other applications. The paper presents the survey of selected methods of time-frequency and time-scale analysis using the Short-Time Fourier Transform (STFT) and Wavelet Transform (WT) at first. Comparison of both methods imply the different and varying resolution in case of the Wavelet transform and it presents some their properties. Signal decomposition enables very important applications in engineering based upon non-linear and non-stationary signal processing including signal de-noising, compression, segmentation and classification. These general methods can be applied to satellite image processing, air pollution analysis and biomedical image processing as well. A special attention is paid to the use of signal processing in signal prediction.

Keywords: Signal Analysis, Signal and Image Processing, Artificial Neural Networks, Discrete Fourier Transform and Wavelet Transform, Signal Prediction

1. INTRODUCTION

Mathematical methods of signal and image analysis and processing form a fundamental tool in many information and control systems based upon sampled data processing. Information technologies assume real data acquisition with a selected sampling period and their following analysis resulting in many applications including signal segmentation, classification, prediction and compression. Similar methods can be applied also in system identification and control.

Methods of signal analysis described in many books and papers (Čížek 1981, Kuc 1982, Box *et al.* 1994, Uhlíř and Sovka 1995, Hayes 1996, Denbigh 1998, Procházka *et al.* 1998, Mulgrew *et al.* 1999) provide a very powerful tools for many applications of digital signal processing and image analysis. In many cases they are based either on the one-dimensional or two-dimensional discrete Fourier transform or on the wavelet transform (Newland 1994, Kingsbury and Magarey 1997). In signal and image processing discussed here, FIR filters are used in many cases. Their further application allows signal components rejection providing a basis for adaptive signal processing and its de-noising as well (Procházka 1993, Provazník 1995).

The paper is devoted to the survey of selected methods of time-frequency and time-scale signal analysis used for signal components detection and classification, to selected topics of linear and non-linear signal modelling allowing its prediction and to basic comments to image processing. Applications include biomedical signals, measurements of energy consumption and environmental signal and image processing. The mathematical description and analysis of such systems is given in the MATLAB/SIMULINK environment (Hicklin and Grace 1993) resulting in algorithms verified for simulated signals at first and applied to real signal processing.

2. SIGNAL ANALYSIS

Methods of time-frequency and time-scale signal analysis form a fundamental tool in signal decomposition, feature extraction, classification and processing including signal de-noising.

2.1 *Discrete Fourier Transform*

In case of an analysis of a signal $\{x(n)\}_{n=0}^{N-1}$ it is possible to apply the Discrete Fourier Transform and namely its fast algorithm (FFT) that enables to find its transform in the form

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\, k\, n\, 2\pi/N} \qquad (1)$$

for $k = 1, \cdots, N$. This transform can be applied either to a selected signal segment or it can be applied in a moving window as a short-time Fourier transform (STFT) allowing non-stationary signal analysis.

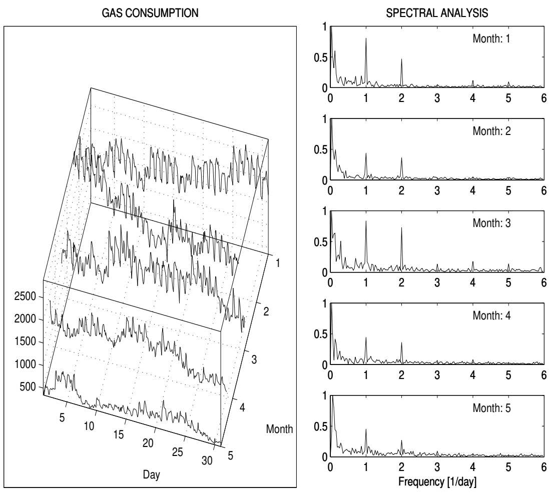

Figure 1. Spectral analysis of 5 given segments each one month long of a time-series representing gas consumption in the Czech Republic measured with the sampling period of two hours during first 5 months of 1997

Fig. 1 presents its use for an analysis of energy (gas) consumption measured with the sampling period of two hours. The time representation of this signal enables to assume the periodic component one day long and the decrease of consumption during Spring months. This assumption can be verified by the FFT that detects periodicity of one day each months presenting higher spectral components as well. This result can be further used in this time series modelling, model order selection and its prediction.

2.2 *Discrete Wavelet Transforms*

Wavelet transforms (WT) provide the alternative to the short-time Fourier transform for non-stationary signal analysis (Strang 1989, Vetterli 1992, Daubechies 1990, Newland 1994). Both STFT and WT result in signal decomposition into two-dimensional function of time and frequency respectively scale that is closely related to frequency. The basic difference between these two transforms is in the construction of the window function which has a constant length in case of the STFT (including rectangular, Blackman and other window functions) while in case of the WT wide windows are applied for low frequencies and short windows for high frequencies to ensure constant time-frequency resolution. Local and global signal analysis can be combined in this way.

Wavelet functions used for signal analysis are derived from the initial basic (mother) function $W(t)$ forming the set of functions

$$\begin{aligned} W_{m,k}(t) &= \frac{1}{\sqrt{a}}\, W\left(\frac{1}{a}\,(t-b)\right) \qquad (2) \\ &= \frac{1}{\sqrt{2^m}}\, W\left(2^{-m} t - k\right) \end{aligned}$$

for discrete parameters of dilation $a = 2^m$ and translation $b = k\, 2^m$.

The basic efficient way to evaluate wavelet transform coefficients using the signal processing notation assumes implementation of the Mallat's pyramidal structure of wavelet transform coefficients evaluation for a given column vector $\{x(n)\}_{n=0}^{N-1}$ according to the scheme given in Fig. 2. Using the signal processing point of view this algorithm assumes the use of the half band low-pass scaling sequence $\{l(n)\}_{n=0}^{L-1}$ together with the corresponding wavelet sequence $\{h(n)\}_{n=0}^{L-1}$ and their convolution with the analyzed signal $\{x(n)\}_{n=0}^{N-1}$ subsampled by two.

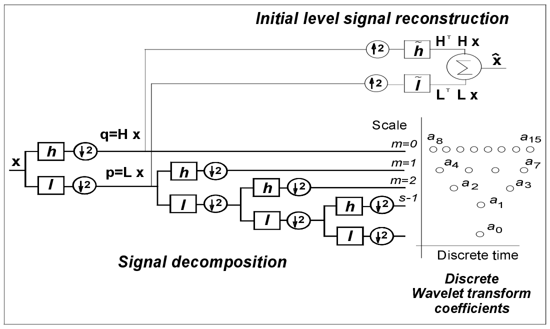

Figure 2. A pyramidal filter bank structure used to evaluate wavelet transform coefficients for a given signal $\{x(n)\}_{n=0}^{N-1}$ and complementary low-pass and high-pass filters $l(n)$ and $h(n)$

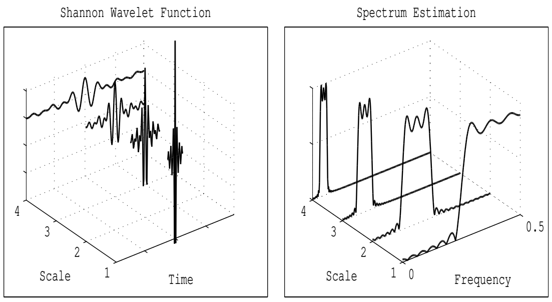

Figure 3. Analysis of the set of Shannon wavelet functions with its initial function in the form $W(t) = \sin(\pi * t/2)/(\pi * t/2) * \cos(3 * \pi * t/2)$ presenting its dilation and the corresponding spectrum compression

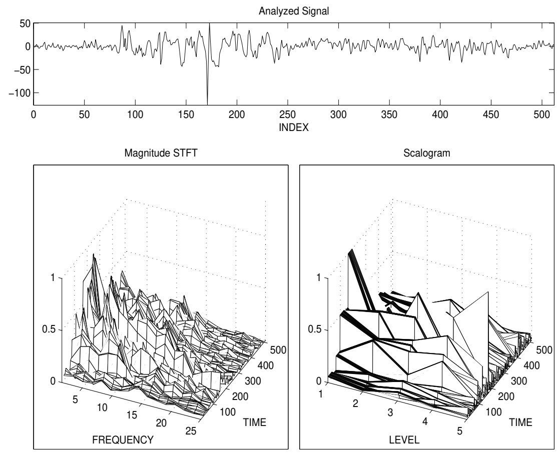

Figure 4. Spectrogram and scalogram of a given data segment representing EEG signal

Various wavelet and scaling sequences can be used for signal analysis defined above. Their dilation correspond to spectrum compression according to Fig. 3. The most common choice include Daubechies wavelets even though their frequency characteristics stand for approximation of band-pass filters only. On the other hand harmonic wavelets introduced in (Newland 1994) can have broader application in many engineering problems owing to their very attractive spectral properties.

Comparison of real EEG signal analysis both by the STFT and WT is given in Fig. 4. Peak detection is much more accurate by the WT comparing with the STFT owing to its changing window size.

Signal decomposition by a given set of wavelet functions into separate levels or scales results in the set of wavelet transform coefficients that can be used for signal compression, signal analysis, segmentation and in case that these coefficients are not modified they allow the following perfect signal reconstruction. In case that only selected levels of signal decomposition are used or wavelet transform coefficients are processed it is possible to extract signal components or to reject its undesirable parts. Using the threshold method introduced by (Strang and Nguyen 1996, Donoho 1995) it is further possible to reject noise and to enlarge signal to noise ratio.

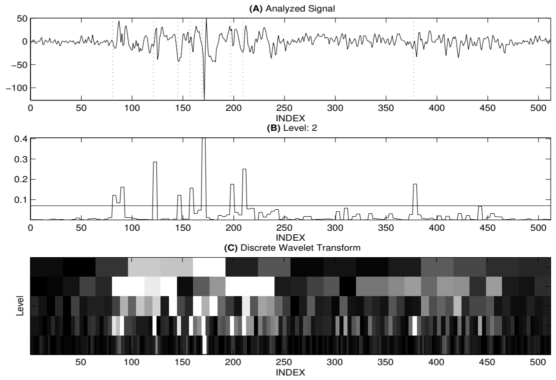

Figure 5. EEG signal decomposition and Wavelet segmentation

The de-noising algorithm assumes that the signal has low frequency components and that it is corrupted by the additive Gaussian white noise with its power much lower than that of the analyzed signal. The whole method consists of the following steps:

- Signal decomposition using a chosen wavelet function up to the selected level and evaluation of wavelet transform coefficients
- The choice of threshold limits for each decomposition level and modification of its coefficients
- Signal reconstruction from modified wavelet transform coefficients

Results of this process depend upon the proper choice of wavelet functions, selection of threshold limits and their use.

The application of threshold limits to modify wavelet coefficients $\{c(k)\}_{k=0}^{N-1}$ include two basic approaches. The use of the soft thresholding formula for a chosen thresholding value δ results in the evaluation of new coefficients

$$\overline{c}_s(k) = \begin{cases} \text{sign } c(k)\ (|c(k)| - \delta) & \text{if } |c(k)| > \delta \\ 0 & \text{if } |c(k)| \leq \delta \end{cases} \quad (3)$$

The hard thresholding method results in the following values of coefficients

$$\overline{c}_h(k) = \begin{cases} \text{sign } c(k) & \text{if } |c(k)| > \delta \\ 0 & \text{if } |c(k)| \leq \delta \end{cases} \quad (4)$$

Method of signal thresholding is given in Fig. 5 for a chosen level of an EEG signal decomposition to find signal segments for its following classification.

3. SIGNAL MODELLING AND PREDICTION

Signal modelling is a very important tool of information engineering with its application in technology, control systems, bioengineering, environmental systems and econometrics. Its theoretical background covers autoregressive models (Ljung 1987) and nonlinear systems based upon artificial neural networks (Haykin 1994) including methods of adaptive signal processing and system optimization.

Methods of signal modelling and prediction include linear and nonlinear systems. While linear autoregressive models are sufficient in many applications including both serially and seasonally related time series it is necessary to use non-linear systems in special cases. The basic neural network structure (Haykin 1994, Procházka and Sýs 1994*b*, Procházka and Sýs 1994*a*) assumes its two layers of sizes $R - S1 - 1$ with a transfer function $F1$ including both sigmoidal and wavelet functions. Output of such a model for any input vector $\mathbf{P}_{R,1}$ is defined by relation

$$\mathbf{Y} = \mathbf{W2} * F1\,(\mathbf{W1} * \mathbf{P} + \mathbf{B1}) + \mathbf{B2} \qquad (5)$$

where matrices $\mathbf{W1}_{S1,R}, \mathbf{W2}_{1,S1}$ define neural network coefficients and vectors $\mathbf{B1}_{S1,1}, \mathbf{B2}_{1,1}$ represent biases.

The basic structure of the adaptive system for signal prediction using the Widrow method (Demuth and Beale 1994) is presented in Fig. 6 assuming a neural network reduced to a linear element only. Each input vector is presented in a sequence to the network causing the adjustment of network weights and biases according to the resulting error. As the linear network is continuously updated it can respond to changes in the relationship between past and future values of a signal. This property is very useful in slowly changing systems using its historical values and model evolution.

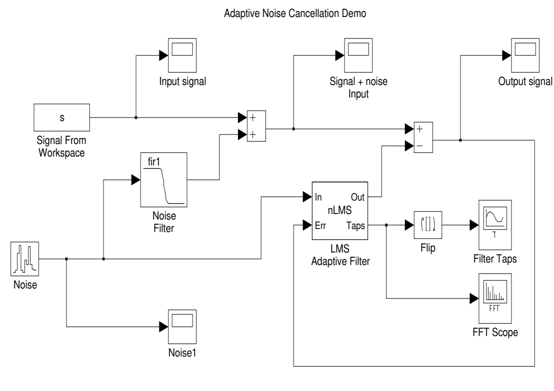

Figure 6. Principle of adaptive signal prediction in the Simulink environment

4. IMAGE PROCESSING

Basic problems of image processing involve their analysis based upon the two-dimensional Fourier transform, their enhancement, segmentation, compression, filtering, de-noising and missing data restoration. Even though it is possible to apply specific methods of image processing it is possible to find that fundamental principles are the same as that used in one-dimensional time series processing.

Both discrete Fourier transform and Wavelet transform are applied in one dimension at first and then in the second one. For a given image matrix $\mathbf{g}_{M,N}$ it is possible to evaluate its two dimensional discrete Fourier transform by relation

$$G_{M,N}(k,l) = \qquad (6)$$
$$= \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} g(m,n) e^{-j\, ln\, 2\pi/N}\ e^{\,-j\, km\, 2\pi/M}$$

for $k = 0, 1, \cdots, M-1$ and $l = 0, 1, \cdots, N-1$. This relation implies that it is possible to apply an algorithm of the discrete Fourier transform twice with rows of the resulting matrix $\mathbf{G}_{M,N}$ standing for frequency components $F_1(k) = k\ 2\pi/M$ and columns allows their interpretation in terms of frequency components $F_2(l) = l\ 2\pi/N$.

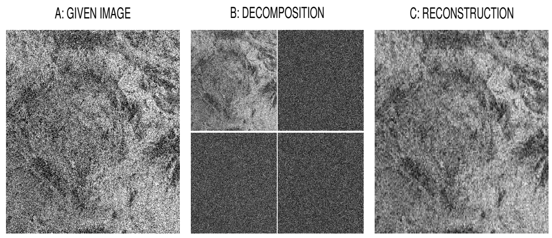

Figure 7. Satellite image of the Czech Republic **(A)** with an additive noise component, **(B)** its decomposition by the Haar wavelet function and **(C)** its reconstruction

In the similar way it is possible to use the Wavelet transform (Kingsbury and Magarey 1997) for signal decomposition. Instead of two one-dimensional time series after the first step of decomposition found in the signal analysis we can fine that in this case we obtain four subimages in four submatrices resulting after the low and high pass convolution of the original matrix by rows and then by columns. Such a decomposition can be used for signal de-noising like in one dimensional case. An example of such a process is presented in Fig. 7 for a satellite image of the Czech Republic with an additional noise.

Correlation of several satellite images can be used in many applications. One of them allows detection of dust particles in the air and their com-

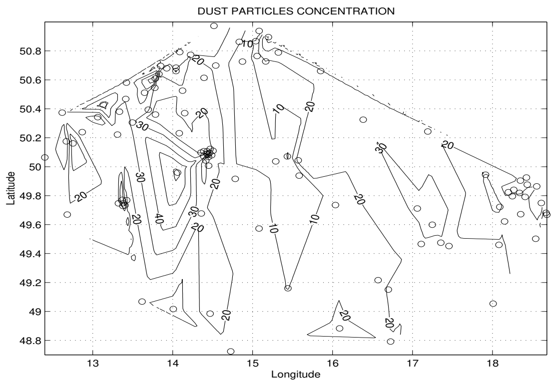

Figure 8. Dust particles concentration over the Czech Republic at a selected time with two-dimensional interpolation from 103 measuring stations

parison with other methods. An example of surface measurements of this component of air pollution is given in Fig. 8 with specification of measuring stations and the following two-dimensional interpolation.

5. CONCLUSION

The paper presents selected methods and application of signal and image processing providing a very efficient tool in many information and control systems. The basic goal of the paper is to summarize selected algorithms, to show that the mathematical background is similar in various research areas and to point to general tools of information engineering and discrete data processing.

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