TWO-DIMENSIONAL FOURIER TRANSFORM IN IMAGE NOISE REJECTION

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Abstract: Fundamental problems of image processing include analysis, noise rejection, edge detection, identification and classifiction of the image data. This paper presents basic mathematical methods and algorithms which can be used for analysis of the image data with the application of the two-dimensional discrete Fourier transform, and its subsequent processing with the application of the two-dimensional Finite Impulse Response (FIR) filter. Design of the convolution kernel for such a filter forms the main part of this paper. Simple methods using moving matrix averaging are compared with sophisticated methods based upon given cutoff frequencies and given precission. Matrix convolution in the time domain is then compared with application of the inverse transform to the multiple of individual transforms in the frequency domain. Additive noise cancelling is first applied to simulated image data to validate the proposed methods. Resulting algorithms are then applied to processing of satellite image data representing given channels of NOAA observations to extract desired information. Further methods of non-linear and adaptive image processing are mentioned as well. All algorithms are presented in the MATLAB environment to provide a clear educational and research tool for further studies.

Keywords: Image Processing, FIR Filters, Discrete Fourier Transform, Convolution, Noise Rejection

1. INTRODUCTION

Methods of signal analysis (Čížek 1981, Uhlíř and Sovka 1995, Kuc 1982) provide very powerful tools in many fields of digital signal processing and image analysis. In many cases these methods are based either on the one-dimensional or two-dimensional discrete Fourier transform or on the wavelet transform (Newland 1994). In signal and image processing discussed here, FIR filters are used. Their further application allows for rejection of signal components providing a basis for adaptive signal processing as well (Procházka 1993, Provazník 1995).

This paper is devoted to use of the two-dimensional FIR filters to design specific systems allowing for noise rejection. The main part of the paper is devoted to the mathematical description and analysis of such a system in the MATLAB/SIMULINK environment (Hicklin and Grace 1993) resulting in an algorithm verified using simulated signals at first and then applied to real signal processing as well, or to image processing (Watkins *et al.* 1993, Chellappa 1992), respectively.

Real image data used in this paper have been supplied by the Czech Hydrometeorological Institute, and they represent satellite images of the Czech Republic. The idea that has led to processing of these images is that they seem to contain information on air pollution, or on the content of solid particles in the air, respectively, and by processing of the data and by applying a suitable filter to them, it may be possible to extract this information.

2. IMAGE PROCESSING

2.1 *The One-Dimensional Fourier Transform*

One method that can be applied to quite any signal processing is the Fourier transform. Its most known one-dimensional form is as follows:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\ kn2\pi/N} \tag{1}$$

for $\{X(k)\}_{k=0}^{N-1}$, where $\{x(n)\}_{n=0}^{N-1}$ is the original signal and $X(k)$ is its Fourier transform. This transform can be applied either to a selected signal segment or it can be applied in the form of a moving window as a Short Time Fourier transform allowing non-stationary signal analysis.

2.2 *The Two-Dimensional Fourier Transform*

For processing of image data the Fourier transform can be used as well, however, its two-dimensional form must be used since here we want to process two-dimensional images. The formula for the two-dimensional Fourier transform is as follows:

$$X(k,l) = \\ = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m,n) e^{-j\ ln2\pi/N} e^{-j\ km2\pi/M} \tag{2}$$

for $k = 0, \ldots, M-1; \quad l = 0, \ldots, N-1$. We can verify correctness of this formula in the following way: Let us assume that our signal to be processed is a simple exponential function of the form

$$x(m,n) = e^{j\ n2\pi/N} = e^{j\ \omega n} \tag{3}$$

producing a two-dimensional matrix $x_{M,N}(m,n)$ where for $n = 0, \ldots N-1, m = 0$ we will have a one-dimensional signal with one frequency ω, and this signal will be repeated in the matrix for all other $m = 1, 2, \ldots M-1$. Applying this exponencial function to our formula for the Fourier transform we obtain

$$X(k,l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} e^{j\ n2\pi/N} e^{-j\ ln2\pi/N} e^{-j\ km2\pi/M}$$

and if we simplify this relationship and use the fact that we can apply the formula for summing of a geometric series, we further obtain

$$X(k,l) = \sum_{m=0}^{M-1} e^{-j\ km2\pi/M} \left(\frac{1 - e^{-j\ n2\pi(l-1)}}{1 - e^{-j\ n2\pi/N(l-1)}} \right)$$

As for evaluation of the expression in parentheses: for l being different from 1 we obtain that the expression is equal to 0, and for l being 1 the expression is equal to N. Thus for l being different from 1 the whole expression for $X(k,l)$ is equal to 0. For l being 1 we can evaluate $X(k,l)$ as follows:

$$X(k,l) = N \sum_{m=0}^{M-1} e^{-j\ km2\pi/M} \tag{4}$$

and applying the formula for summing of a geometric series once again we obtain

$$X(k,l) = N \frac{1 - e^{-j\ km2\pi}}{1 - e^{-j\ km2\pi/M}} \tag{5}$$

For k being different from 0 we find out using the Euler formulas that the resulting value of $X(k,l)$ is 0; for $k = 0$ we must apply the l'Hospital rule, and we find out that in this case $X(k,l) = N$. We can thus conclude that the original signal (3) contains one frequency component only which is correct, and that our original formula for the two-dimensional Fourier transform (2) is correct as well. Application of this method is shown in Figs. 1a), 2a), and 3a). for simulated data, and for frequencies $F1 = 2\pi/N$, and $F2 = 2\pi/M$.

2.3 *The Two-Dimensional FIR Filter*

It is a commonly known fact that FIR filters can be used to modify frequency components of the original data according to our specifications. As here we deal with two-dimensional data, we must use a two-dimensional filter. Assuming we have the original (noisy) image matrix $x(m,n)$, its form can be found in the following way:

$$y(m,n) = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} h(k,l) x(m-k, n-l) \tag{6}$$

where $y(m,n)$ denotes the filter output, for the convolution matrix

$$H_{K,L} = \begin{bmatrix} h(0,0) & \ldots & h(0,L-1) \\ \vdots & \ddots & \vdots \\ h(K-1,0) & \ldots & h(K-1,L-1) \end{bmatrix}$$

Let us assume we have a given two-dimensional signal $x(m,n)$ containing two frequency components ω_1, ω_2:

$$x(m,n) = e^{j\omega_1 m} e^{j\omega_2 n} \tag{7}$$

Output of linear systems has the same frequency components as the input, modified in the amplitude and phase

$$y(m,n) = H(e^{j\omega_1 m}, e^{j\omega_2 n}) e^{j\omega_1 m} e^{j\omega_2 n} \quad (8)$$

After substituting Eqns. (7) and (8) into (6), we obtain the system frequency response in the form

$$H(e^{j\omega_1}, e^{j\omega_2}) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} h(m,n) e^{-j\omega_1 m} e^{-j\omega_2 n}$$

In the case the desired frequency response has the following form

$$\mathrm{H}(\omega_1,\omega_2) = \begin{cases} e^{-j\,\omega_1\alpha_1} e^{-j\,\omega_2\alpha_2} & for: \\ & -\omega_{1c} < \omega_1 < \omega_{1c} \\ & -\omega_{2c} < \omega_2 < \omega_{2c} \\ 0 & elsewhere \end{cases}$$

we can find the filter coefficients applying the two-dimensional Fourier series in the form

$$h(m,n) = \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} H(\omega_1,\omega_2) e^{j\omega_1 k} e^{j\omega_2 l} d\omega_1 d\omega_2$$

$$= \frac{1}{(2\pi)^2} \int_{-\omega_{2c}}^{\omega_{2c}} \int_{-\omega_{1c}}^{\omega_{1c}} e^{-j\omega_1\alpha_1} e^{-j\omega_2\alpha_2} e^{j\omega_1 k} e^{j\omega_2 l} d\omega_1 d\omega_2$$

And we find the following solution for the coefficients

$$h(m,n) = \frac{\sin\omega_{1c}(m-\alpha_1)}{m-\alpha_1} \; \frac{\sin\omega_{2c}(n-\alpha_2)}{n-\alpha_2} \quad (9)$$

where $\alpha_1 = \frac{M-1}{2}$ and $\alpha_2 = \frac{N-1}{2}$.

This filter has been applied to the simulated (see Figs. 4 and 5) as well as real satellite data, and other filters have been used which are incorporated in the MATLAB program, and whose description can be found in related MATLAB manuals.

3. APPLICATIONS AND RESULTS

3.1 *Simulated Data*

We have tested application of the two-dimensional Fourier transform on simulated data at first, see Figs. 1, 2, and 3, and then we have applied a FIR filter to the given simulated image, and the results of this are given in Figs. 4 and 5. The noisy image presented in Fig. 1 b) can be processed in this way to obtain the final image shown in Fig. 4 b) which is very similar to the original one shown in Fig. 1 a).

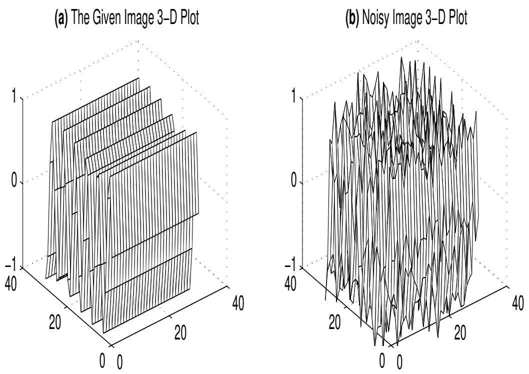

Figure 1. The 3-D plot of simulated data $x(m,n) = \sin(12\pi/31n), m = 1,\ldots,40; n = 1,\ldots,40$; Part a) shows the original data, part b) shows the data with added high frequency noise

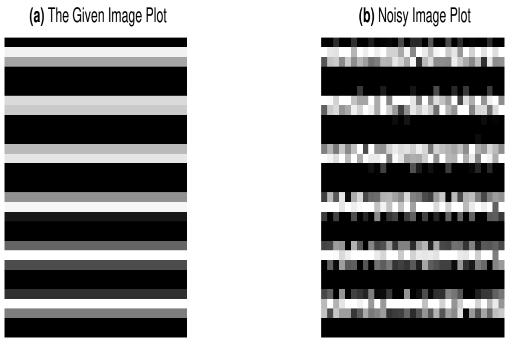

Figure 2. Image plots of simulated matrices presented in Fig. 1

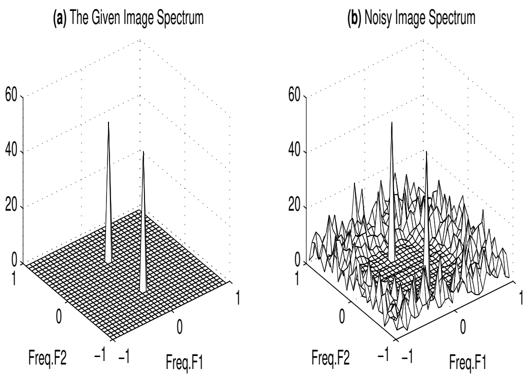

Figure 3. The visualization of the two-dimensional Fourier transform: in part a) it is applied to the pure original simulated data given in Fig. 1, in part b) it is applied to the same data with high frequency components added. It can be seen from both parts that using this method we can clearly identify the one frequency (here denoted as $F2$) present in the original data

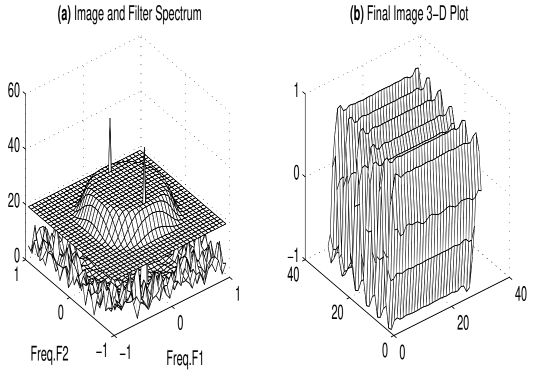

Figure 4. The application of the FIR filter to simulated data; in part a) the image and filter spectrum is given obtained using the two-dimensional Fourier transform, in part b) the final denoised image is shown as a 3-D plot

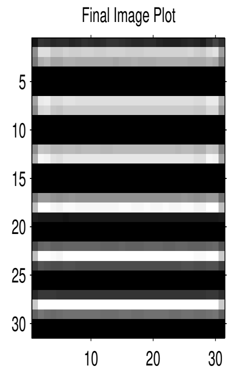

Figure 5. The final denoised image obtained after application of the FIR filter

3.2 *Satellite Data*

The two-dimensional Fourier transform and the FIR filter have been applied to data supplied by the Czech Hydrometeorological Institute. These are satellite data showing the surface of the Czech Republic. These data are used for weather forecasts, and they may be possibly used for estimation of air pollution which is the purpose of their processing at our department. Fig. 6 shows a sample satellite image with artificial noise added, and Fig. 7 shows the same image denoised using the FIR filter.

4. CONCLUSION

Algorithms described in this paper present methods of noise removal both in the case of a simulated signal and of real satellite images. The methods applied have given very good results and they will be further applied to extracting of information on air pollution from the satellite data.

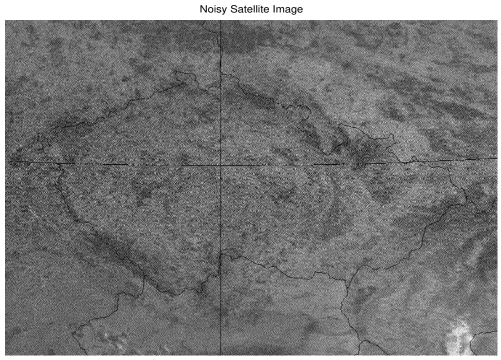

Figure 6. Satellite image of the Czech Republic with artificial noise added

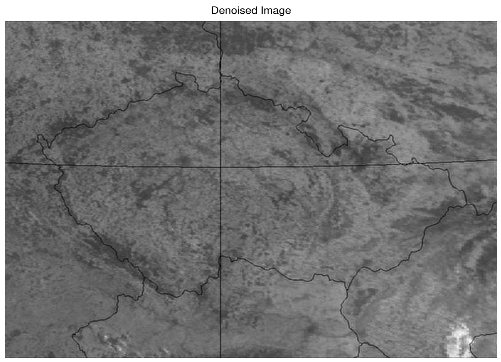

Figure 7. Denoised satellite image using the FIR filter

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